

Matemáticas II
2º Bachillerato
Capítulo 10: Integrales

Respuestas a los ejercicios y problemas propuestos

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IES ATENEA, CIUDAD REAL

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Actividades propuestas

1. Calcula las siguientes primitivas

$$\text{a) } \int 4x^3 dx \quad \text{b) } \int 3x^2 dx \quad \text{c) } \int 5x^4 dx \quad \text{d) } \int (5x^4 - 4x^3 + 3x^2) dx$$

$$\text{a) } \int 4x^3 dx = x^4 + C$$

$$\text{b) } \int 3x^2 dx = \frac{3x^3}{3} + C = x^3 + C$$

$$\text{c) } \int 5x^4 dx = \frac{5x^5}{5} + C = x^5 + C$$

$$\text{d) } \int (5x^4 - 4x^3 + 3x^2) dx = x^5 - x^4 + x^3 + C$$

2. Dada $f(x) = (x^3 - 3x^2 + 2x + 1)$, calcula la primitiva $F(x)$ de $f(x)$ que verifica $F(0) = 4$

$$F(x) = \int (x^3 - 3x^2 + 2x + 1) dx = \frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} + x + C = \frac{x^4}{4} - x^3 + x^2 + x + C$$

Como $F(0) = 4$, $F(0) = \frac{0^4}{4} - 0^3 + 0^2 + 0 + C = 4$, luego $C = 4$, de donde,

$$F(x) = \frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} + x + 4$$

3. Comprueba si $F(x) = (4x^3 + 2x^2 - x + 5)$ es una primitiva de $f(x) = (12x^2 + 4x + 3)$. En caso negativo, explica por qué.

La derivada de $F(x)$ ha de ser igual a $f(x)$

Si derivamos $F(x)$ obtenemos $F'(x) = 12x^2 + 4x - 1 \neq f(x)$

Por tanto, $F(x)$ no es una primitiva de $f(x)$

4. Determina los valores de a , b , c y d para los que $(4a^3 + bx^2 + cx + d)$ es una primitiva de la función $f(x) = (4x^2 - 5x + 3)$

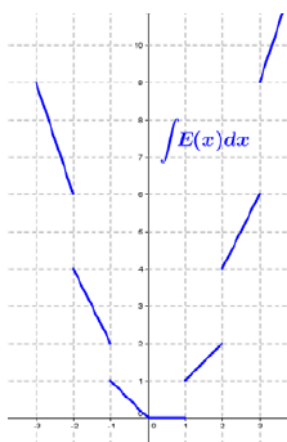
$$F(x) = \int (4x^2 - 5x + 3) dx = 4\frac{x^3}{3} - 5\frac{x^2}{2} + 3x + C, \text{ por tanto,}$$

$$a = \frac{4}{3}, \quad b = \frac{-5}{2}, \quad c = 3, \quad d = C$$

5. Al resolver una primitiva, Javier y Ricardo han utilizado métodos diferentes y, como era de esperar, han obtenido expresiones distintas. Después de revisarlo muchas veces y no encontrar ningún error en los cálculos, le llevan el problema a la profesora para ver quien tiene bien el ejercicio. Para su sorpresa, la profesora les dice que ambos tienen bien el problema. ¿Cómo es posible?

Pueden diferir en constantes y estar los dos bien, además de las posibles simplificaciones.

6. Razona por qué la gráfica siguiente:



es una primitiva de la función “parte entera de x ”, $E(x)$, (salvo en los puntos de discontinuidad donde no es derivable):

$$\int 1 dx = x \quad \int 2 dx = 2x \quad \int 3 dx = 3x \quad \int (-1) dx = -x \quad \int (-2) dx = -2x$$

7. Calcula las siguientes primitivas utilizando el cambio indicado:

a) $\int \frac{\sqrt{x} - \sqrt[3]{x}}{\sqrt[4]{x}} dx$ haciendo $x = t^{12}$.

$x = t^{12}$, $dx = 12t^{11} dt$, sustituyendo, obtenemos $\int \frac{\sqrt{t^{12}} - \sqrt[3]{t^{12}}}{\sqrt[4]{t^{12}}} \cdot 12t^{11} dt$, simplificando,

$$\int \frac{t^6 - t^4}{t^3} 12t^{11} dt = \int (t^6 - t^4) 12t^8 dt = 12 \int (t^{14} - t^{12}) dt = 12 \left(\frac{t^{15}}{15} - \frac{t^{13}}{13} \right) + C$$

$$x = t^{12}, \quad t = \sqrt[12]{x}, \quad \text{deshaciendo el cambio, } \int \frac{\sqrt{x} - \sqrt[3]{x}}{\sqrt[4]{x}} \cdot dx = 12 \left(\frac{(\sqrt[12]{x})^{15}}{15} - \frac{(\sqrt[12]{x})^{13}}{13} \right) + C$$

b) $\int \frac{dx}{e^x + e^{-x}}$ haciendo $e^x = t$.

$e^x = t$, $e^x dx = dt$, $dx = \frac{dt}{e^x} = \frac{dt}{t}$, $e^{-x} = \frac{1}{e^x} = \frac{1}{t}$ sustituyendo,

$$\int \frac{dx}{e^x + e^{-x}} = \int \frac{\frac{dt}{t}}{\frac{t}{t} + \frac{1}{t}} = \int \frac{\frac{dt}{t}}{\frac{t^2 + 1}{t}} = \int \frac{dt}{t^2 + 1} = \arctg(t) + C = \arctg(e^x) + C$$

c) $\int \frac{5x^4}{\sqrt{1+2x}} dx$ haciendo $1+2x = t^2$

$$1 + 2x = t^2, \quad 2dx = 2t dt, \quad dx = t dt, \quad x = \frac{t^2 - 1}{2}, \quad t = \sqrt{1 + 2x}$$

$$\int \frac{5x^4}{\sqrt{1+2x}} dx = \int \frac{5 \left(\frac{t^2 - 1}{2} \right)^4}{\sqrt{t^2}} t dt = 5 \int \frac{(t^2 - 1)^4}{2^4} dt = \frac{5}{16} \int (t^8 - 4t^6 + 6t^4 - 4t^2 + 1) dt =$$

$$= \frac{5}{16} \left(\frac{t^9}{9} - 4 \frac{t^7}{7} + 6 \frac{t^5}{5} - 4 \frac{t^3}{3} + t \right) + C =$$

$$= \frac{5}{16} \left(\frac{(\sqrt{1+2x})^9}{9} - 4 \frac{(\sqrt{1+2x})^7}{7} + 6 \frac{(\sqrt{1+2x})^5}{5} - 4 \frac{(\sqrt{1+2x})^3}{3} + \sqrt{1+2x} \right) + C$$

d) $\int \frac{dx}{x + \sqrt{x^2 - 1}}$ haciendo $x + \sqrt{x^2 - 1} = t$

$$x + \sqrt{x^2 - 1} = t, \quad \sqrt{x^2 - 1} = t - x, \quad (\sqrt{x^2 - 1})^2 = (t - x)^2,$$

$$x^2 - 1 = t^2 + x^2 - 2xt, \quad 2xt = t^2 + 1, \quad x = \frac{t^2 + 1}{2t},$$

$$dx = \frac{2t \cdot 2t - (t^2 + 1) \cdot 2}{4t^2} dt = \frac{(2t^2 - 2)}{4t^2} dt = \frac{t^2 - 1}{2t^2} dt, \text{ de donde,}$$

$$\int \frac{dx}{x + \sqrt{x^2 - 1}} = \int \frac{1}{t} \cdot \frac{t^2 - 1}{2t^2} dt = \frac{1}{2} \int \left(\frac{t^2}{t^3} - \frac{1}{t^3} \right) dt = \frac{1}{2} \int \left(\frac{1}{t} - t^{-3} \right) dt = \frac{1}{2} \left(\ln|t| + \frac{t^{-2}}{2} \right) + C$$

$$\text{Deshaciendo el cambio, } \int \frac{dx}{x + \sqrt{x^2 - 1}} = \frac{1}{2} \left(\ln|x + \sqrt{x^2 - 1}| + \frac{1}{2(x + \sqrt{x^2 - 1})^2} \right) + C$$

e) $\int (2 \sin^3 x + 3 \sin^2 x - \sin x + 3) \cos x \, dx$ haciendo $\sin x = t$

$\sin x = t$, $\cos x \, dx = dt$, sustituyendo, nos queda,

$$\int (2t^3 + 3t^2 - t + 3) dt = 2 \frac{t^4}{4} + 3 \frac{t^3}{3} - \frac{t^2}{2} + 3t + C = \frac{\sin^4 x}{2} + \sin^3 x - \frac{\sin^2 x}{2} + 3 \sin x + C$$

f) $\int \sqrt{1 - x^2} \, dx =$ Haciendo $x = \sin t$; $t = \arcsin x$; $dx = \cos t \, dt$

$$\int \sqrt{1 - \sin^2 t} \cos t \, dt = \int \sqrt{\cos^2 t} \cdot \cos t \, dt = \int \cos t \cdot \cos t \, dt = \int \cos^2 t \, dt = \int \frac{1 + \cos 2t}{2} dt =$$

$$= \int \frac{1}{2} dt + \frac{1}{2} \int \cos 2t \, dt = \int \frac{1}{2} dt + \frac{1}{2} \cdot \frac{1}{2} \int 2 \cos 2t \, dt = \frac{1}{2} t + \frac{1}{4} \cdot \sin 2t = \frac{1}{2} \arcsin x + \frac{1}{4} x + C$$

8. Elige el cambio que simplifica las siguientes integrales:

a) $\int \frac{2x^3 + 1}{(x^4 + 2x)^3} dx$

$$t = x^4 + 2x, \quad dt = (4x^3 + 2) dx = 2(2x^3 + 1) dx$$

b) $\int \frac{e^{\tan x}}{\cos^2 x} dx$

$$t = \tan x, \quad dt = \frac{1}{\cos^2 x} dx$$

c) $\int \frac{\ln(\ln x)}{x \cdot \ln x} dx$

$$t = \ln(\ln x), \quad dt = \frac{1}{x \ln x} dx = \frac{1}{x \ln x} dx$$

$$\begin{aligned} \text{d) } \int 2x^3 \sqrt{x^4 - 49} \cdot dx & \quad t = x^4 - 49, \quad dt = 4x^3 dx \\ \text{e) } \int \frac{x+1}{\sqrt[3]{x+1}+2} dx & \quad t^3 = x+1, \quad 3t^2 dt = dx \\ \text{f) } \int \frac{x}{\sqrt{1-4x^2}} dx & \quad t = 1-4x^2, \quad dt = -8x dx \end{aligned}$$

9. Determina si las siguientes integrales son inmediatas o no:

$$\begin{aligned} \text{a) } \int \left(4x^3 + 3x^3 - \frac{1}{x^2} + \sqrt{x} \right) dx & \quad \text{Sí.} \\ \text{b) } \int \frac{\ln x}{x} dx & \quad \text{Sí} \quad \left(\int \frac{\ln x}{x} dx = \int \ln x \cdot \frac{1}{x} dx \right) \\ \text{c) } \int \sin x \cos x dx & \quad \text{Sí} \\ \text{d) } \int \frac{e^{\arcsen x} dx}{\sqrt{1-x^2}} = e^{\arcsen x} + C & \quad \text{Sí} \\ \text{e) } \int \frac{\arctg x}{1+x^2} dx = \frac{(\arctg x)^2}{2} + C & \quad \text{Sí} \\ \text{f) } \int \frac{\ln(x+1)}{x} dx & \quad \text{NO} \\ \text{g) } \int \lg x \cdot \cos x dx = \int \frac{\sen x}{\cos x} \cdot \cos x dx = \int \sen x dx = -\cos x + C & \quad \text{Sí} \\ \text{h) } \int \frac{x^2-1}{\sqrt{1-x^2}} dx & \quad \text{Sí} \\ \text{i) } \int e^{x^2} dx & \quad \text{NO} \\ \text{k) } \int \frac{x^4-2x^2+1}{x^2-1} dx & \quad \text{Sí} \quad [x^4-2x^2+1 = (x^2-1)^2] \\ \text{j) } \int x^2 \cdot e^{x^2} dx & \quad \text{NO} \end{aligned}$$

10. Resuelve las siguientes integrales:

$$\begin{aligned} \text{a) } \int (e^{3x} + e^{2x} + e^x) e^x dx, \quad t = e^x, dt = e^x dx, \int (t^3 + t^2 + t) dt &= \frac{t^4}{4} + \frac{t^3}{3} + \frac{t^2}{2} + C = \frac{e^{4x}}{4} + \frac{e^{3x}}{3} + \frac{e^{2x}}{2} + C \\ \text{b) } \int (\ln x + 2) \frac{dx}{x} &= \frac{(\ln x + 2)^2}{2} + C \end{aligned}$$

$$\begin{aligned}
 \text{c) } \int \ln(\cos x) \operatorname{tg} x \, dx, \quad t = \ln(\cos x), \quad dt = -\frac{\operatorname{sen} x}{\cos x} dx = -\operatorname{tg} x dx = \\
 = -\int t dt = -\frac{t^2}{2} + C = -\frac{(\ln(\cos x))^2}{2} + C \\
 \text{d) } \int \frac{x \, dx}{1+x^4} = \frac{1}{2} \int \frac{2x}{1+(x^2)^2} dx = \frac{1}{2} \operatorname{arctg}(x^2) + C \\
 \text{i) } \int \frac{e^x dx}{1+e^{2x}} = \int \frac{e^x}{1+(e^x)^2} dx = \operatorname{arctg}(e^x) + C \\
 \text{j) } \int x \cdot \cos e^{x^2} \cdot e^{x^2} dx, \quad t = e^{x^2}, \quad dt = 2xe^{x^2} dx, \quad \frac{1}{2} \int \cos t dt = \frac{1}{2} \operatorname{sen} t + C = \frac{1}{2} \operatorname{sen}(e^{x^2}) + C
 \end{aligned}$$

11. Resuelve las siguientes integrales:

$$\begin{aligned}
 \text{a) } \int (x^2 + x + 1)e^x dx \quad \left\{ \begin{array}{l} u = x^2 + x + 1 \rightarrow du = (2x + 1)dx \\ dv = e^x dx \rightarrow v = \int e^x dx = e^x \end{array} \right\} \\
 \int (x^2 + x + 1)e^x dx = (x^2 + x + 1)e^x - \int (2x + 1)e^x dx = \\
 \left\{ \begin{array}{l} u = 2x + 1 \rightarrow du = 2dx \\ dv = e^x dx \rightarrow v = \int e^x dx = e^x \end{array} \right\} = (x^2 + x + 1)e^x - [(2x + 1)e^x - \int 2e^x dx] = \\
 = (x^2 + x + 1)e^x - (2x + 1)e^x + 2e^x + C = (x^2 - x + 2)e^x + C \\
 \text{b) } \int \ln x \, dx \quad \left\{ \begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = dx \rightarrow v = \int dx = x \end{array} \right\} \quad \int \ln x dx = \ln|x|x - \int x \frac{1}{x} dx = x \ln|x| - x + C \\
 \text{c) } \int x \cos x dx \quad \left\{ \begin{array}{l} u = x \rightarrow du = dx \\ dv = \cos x dx \rightarrow v = \int \cos x dx = \operatorname{sen} x \end{array} \right\} \\
 \int x \cos x dx = x \operatorname{sen} x - \int \operatorname{sen} x dx = x \operatorname{sen} x + \cos x + C
 \end{aligned}$$

Curiosidad – idea feliz: Resuelve la primitiva $\int \cos(\ln x) dx$.

Para ello, multiplica y divide el integrando por x : $\int \frac{\cos(\ln x)}{x} \cdot x dx = \left| \begin{array}{l} u = x \rightarrow du = \dots \\ dv = \frac{\cos(\ln x)}{x} dx \rightarrow v = \dots \end{array} \right|$

$$\begin{aligned}
 I = \int \cos(\ln x) dx &= \int \frac{\cos(\ln x)}{x} \cdot x dx = \left\{ \begin{array}{l} u = x \rightarrow du = dx \\ dv = \frac{\cos(\ln x)}{x} dx \rightarrow v = \operatorname{sen}(\ln x) \end{array} \right\} \\
 &= x \operatorname{sen}(\ln x) - \int \operatorname{sen}(\ln x) dx.
 \end{aligned}$$

$$\int \operatorname{sen}(\ln x) dx = \int \frac{\operatorname{sen}(\ln x)}{x} \cdot x dx = \left\{ \begin{array}{l} u = x \rightarrow du = dx \\ dv = \frac{\operatorname{sen}(\ln x)}{x} dx \rightarrow v = -\cos(\ln x) \end{array} \right\}$$

$$= -x \cos(\ln x) + \int \cos(\ln x) dx.$$

$$I = x \sin(\ln x) + x \cos(\ln x) - I, \quad 2I = x \sin(\ln x) + x \cos(\ln x)$$

$$I = \frac{1}{2} x (\sin(\ln x) + \cos(\ln x)) + C$$

$$\begin{aligned} \text{d) } \int \arcsen x dx &= \left[\begin{array}{l} u = \arcsen x \rightarrow du = \frac{1}{\sqrt{1-x^2}} dx \\ dv = 1 dx \rightarrow v = x \end{array} \right] = \\ \arcsen x \cdot x - \int \frac{x}{\sqrt{1-x^2}} dx &= \left[t = 1-x^2 \rightarrow dt = -2x dx \rightarrow \frac{dt}{-2} = x dx \right] \\ \arcsen x \cdot x - \int \frac{\frac{dt}{-2}}{\sqrt{t}} &= \arcsen x \cdot x + \frac{1}{2} \int t^{-\frac{1}{2}} dt = \arcsen x \cdot x + \frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} = \\ &= \arcsen x \cdot x + t^{\frac{1}{2}} = \arcsen x \cdot x + \sqrt{1-x^2} + C \end{aligned}$$

$$\begin{aligned} \text{e) } \int \sen ax \cdot e^{bx} dx &= \left[\begin{array}{l} u_1 = \sen(ax) \rightarrow du_1 = a \cdot \cos(ax) dx \\ dv_1 = e^{bx} dx \rightarrow v_1 = \frac{e^{bx}}{b} \end{array} \right] \\ = \frac{e^{bx} \sen(ax)}{b} - \int \frac{a e^{bx} \cos(ax)}{b} dx &= \left[\begin{array}{l} u_2 = a \cdot \cos(ax) \rightarrow du_2 = -a^2 \sen(ax) dx \\ dv_1 = \frac{e^{bx}}{b} \rightarrow v_1 = \frac{e^{bx}}{b^2} \end{array} \right] \\ = \frac{e^{bx} \sen(ax)}{b} - \left(\frac{a e^{bx} \cos(ax)}{b^2} + \int \frac{a^2 e^{bx} \sen(ax)}{b^2} dx \right) &= \\ = \frac{e^{bx} \sen(ax)}{b} - \left(\frac{a e^{bx} \cos(ax)}{b^2} + \frac{a^2}{b^2} \int \sen ax \cdot e^{bx} dx \right) &= \\ \frac{b e^{bx} \sen(ax) - a e^{bx} \cos(ax)}{b^2} - \frac{a^2}{b^2} \int \sen ax \cdot e^{bx} dx &= \\ \text{Haciendo } I = \int \sen ax \cdot e^{bx} dx &= \\ I = \frac{b e^{bx} \sen(ax) - a e^{bx} \cos(ax)}{b^2} - \frac{a^2}{b^2} I &= \\ I + \frac{a^2}{b^2} I = \frac{b e^{bx} \sen(ax) - a e^{bx} \cos(ax)}{b^2} ; \frac{a^2 + b^2}{b^2} I = \frac{b e^{bx} \sen(ax) - a e^{bx} \cos(ax)}{b^2} &= \\ \text{De donde, } I = \frac{b e^{bx} \sen(ax) - a e^{bx} \cos(ax)}{a^2 + b^2} + C &= \end{aligned}$$

12. Resuelve las siguientes primitivas.

$$\text{a) } \int \frac{dx}{x^2-4} \quad x^2 - 4 = 0; \quad x_1 = 2, x_2 = -2$$

$$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2)+B(x-2)}{(x^2-4)} \rightarrow 1 = A(x+2) + B(x-2)$$

$$x = 2; \quad 1 = 4A \rightarrow A = \frac{1}{4}$$

$$x = -2; \quad 1 = -4B \rightarrow B = -\frac{1}{4}$$

$$\int \frac{dx}{x^2-4} = \int \frac{A}{(x-a)} dx + \int \frac{B}{(x-b)} dx = \int \frac{1/4}{x-2} dx + \int \frac{-1/4}{x+2} dx =$$

$$= \frac{1}{4} \int \frac{1}{x-2} dx - \frac{1}{4} \int \frac{1}{x+2} dx = \frac{1}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| = \frac{1}{4} (\ln|x-2| - \ln|x+2|) = \frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + C$$

b) $\int \frac{dx}{(x+1)^2}$

$$\int \frac{dx}{(x+1)^2} = \int \frac{1}{(x+1)^2} dx = \int (x+1)^{-2} dx = \frac{(x+1)^{-1}}{-1} = \frac{-1}{x+1} + C$$

c) $\int \frac{x dx}{(x+1)^2}$

$$(x+1)^2 = 0; \quad x_1 = -1, x_2 = -1$$

$$\frac{x}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2} \rightarrow x = A(x+1) + B$$

$$x = -1; \quad B = -1$$

$$x = 0; \quad A = 1$$

$$\int \frac{x dx}{(x+1)^2} = \int \frac{1}{x+1} dx + \int \frac{-1}{(x+1)^2} dx = \int \frac{1}{x+1} dx - \int (x+1)^{-2} dx =$$

$$= \ln|x+1| - \frac{(x+1)^{-1}}{-1} = \ln|x+1| + \frac{1}{x+1} + C$$

d) $\int \frac{x^3}{(x+1)^2} dx$, efectuamos la división, obtenemos $C(x) = x - 2$; $R(x) = 3x + 2$, luego

$$\int \frac{x^3}{(x+1)^2} dx = \int (x-2) dx + \int \frac{3x+2}{x^2+2x+1} dx, \text{ como } x^2 + 2x + 1 = (x+1)^2$$

$$\int \frac{x^3}{(x+1)^2} dx = \int (x-2) dx + \frac{3}{2} \int \frac{2x}{x^2+2x+1} dx + 2 \int \frac{1}{(x+1)^2} dx =$$

$$\frac{x^2}{2} - 2x + \frac{3}{2} \ln|x^2 + 2x + 1| - \frac{2}{x+1} + C$$

e) $\int \frac{x^2+x+1}{x^3-4x^2+4x} dx = \int \frac{x^2+x+1}{(x^2-4x+4)(x)} dx = \int \frac{x^2+x+1}{(x)(x-2)^2} dx$

$$\frac{x^2+x+1}{(x)(x-2)^2} = \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$x^2 + x + 1 = A(x-2)^2 + B(x)(x-2) + C(x)$$

$$x = 0; \quad 1 = 4A \rightarrow A = \frac{1}{4}$$

$$x = 2; \quad 7 = 2C \rightarrow C = \frac{7}{2}$$

$$x = 1; \quad 3 = \frac{1}{4} - B + \frac{7}{2} \rightarrow \frac{-3}{4} = -B \rightarrow \frac{3}{4} = B$$

$$\int \frac{x^2+x+1}{(x)(x-2)^2} dx = \frac{1}{4} \int \frac{1}{x} dx + \frac{3}{4} \int \frac{1}{x-2} dx + \frac{7}{2} \int \frac{1}{(x-2)^2} dx \rightarrow \frac{1}{4} \ln|x| + \frac{3}{4} \ln|x-2| - \frac{7}{2} \left(\frac{1}{x-2} \right) + C$$

$$f) \int \frac{3x^2+1}{(2x-1)(3x^2+2)} dx; \frac{3x^2+1}{(2x-1)(3x^2+2)} = \frac{A}{2x-1} + \frac{Mx+N}{3x^2+2}$$

$$\rightarrow 3x^2 + 1 = A(3x^2 + 2) + (Mx + N)(2x - 1)$$

$$x = 0 \rightarrow 1 = 2A - N \rightarrow N = 2A - 1 \rightarrow 2\left(\frac{7}{11}\right) - 1 \rightarrow N = \frac{3}{11}$$

$$x = 1 \rightarrow 4 = A(5) + (M + N) \rightarrow 4 = 5A + M + N \rightarrow 4 = 5A + M + 2A - 1 \rightarrow M = \frac{6}{11}$$

$$x = 2 \rightarrow 13 = 14A + (2M + N)3 \rightarrow 13 = 14A + 6M + 3N \rightarrow A = \frac{7}{11}$$

$$\begin{aligned} \int \frac{3x^2+1}{(2x-1)(3x^2+2)} dx &= \frac{7}{11} \int \frac{1}{2x-1} dx + \int \frac{6x+3}{11(3x^2+2)} dx = \frac{7}{22} \int \frac{2}{2x-1} dx + \frac{1}{11} \int \frac{6x+3}{3x^2+2} dx = \\ &= \frac{7}{22} \ln|2x-1| + \frac{1}{11} \int \frac{6x+3}{3x^2+2} dx = \frac{7}{22} \ln|2x-1| + \frac{1}{11} \int \frac{6x}{3x^2+2} dx + \frac{3}{11} \int \frac{\sqrt{3}}{(\sqrt{3}x)^2+2} dx = \\ &= \frac{7}{22} \ln|2x-1| + \frac{1}{11} \ln|3x^2+2| + \frac{3}{11} \cdot \frac{1}{\sqrt{2}} \arctan\left(\frac{\sqrt{3}x}{\sqrt{2}}\right) + C \end{aligned}$$

$$g) \int \frac{x^2-2}{x^3(x^2+1)} dx \rightarrow \frac{x^2-2}{x^3(x^2+1)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{Dx+E}{x^2+1} \rightarrow$$

$$\rightarrow x^2 - 2 = A(x^2 + 1) + B(x(x^2 + 1)) + Cx^2(x^2 + 1) + (Dx + E)x^3 \rightarrow$$

$$x = 0; -2 = A$$

$$\begin{cases} x = -1; -1 = -4 - 2B + 2C - (-D + E) \\ x = 1; -1 = -4 + 2B + 2C + D + E \\ x = 2; 2 = -10 + 10B + 20C + 8(2D + E) \\ x = -2; 2 = -10 - 10B + 20C - 8(-2D + E) \end{cases} \rightarrow F_1 + F_2 \rightarrow 5F_1 + F_3 \rightarrow -5F_1 + F_4$$

$$\rightarrow \begin{cases} 3 = -2B + 2C + D - E \\ 6 = 4C + 2D \\ 27 = 30C + 21D + 3E \\ -3 = 10C + 11D - 3E \end{cases} \rightarrow B = 0; C = 3; D = -3; E = 0 \rightarrow$$

$$\begin{aligned} \int \frac{x^2-2}{x^3(x^2+1)} dx &= \int \frac{-2}{x^3} dx + \int \frac{3}{x} dx + \int \frac{-3x}{x^2+1} dx = -2 \int \frac{1}{x^3} dx + 3 \int \frac{1}{x} dx - \frac{3}{2} \int \frac{2x}{x^2+1} dx = \\ &= -2 \frac{x^{-2}}{-2} + 3 \ln|x| - \frac{3}{2} \ln|x^2 + 1| + C = \frac{1}{x^2} + 3 \ln|x| - \frac{3}{2} \ln|x^2 + 1| + C \end{aligned}$$

$$h) \int \frac{x^3+2x^2+5x+3}{x^2+1} dx = (x^3 + 2x^2 + 5x + 3):(x^2 + 1); \quad r(x) = 4x + 1; \quad c(x) = x + 2$$

$$= \int (x + 2 + \frac{4x+1}{x^2+1}) dx = \int x dx + \int 2 dx + \int \frac{4x}{x^2+1} dx + \int \frac{1}{x^2+1} dx =$$

$$= \frac{x^2}{2} + 2x + 2 \ln(x^2 + 1) + \arctg(x) + C$$

$$i) \int \frac{x+1}{(x-1)(x+1)^2(x^2+1)} dx \rightarrow \frac{x+1}{(x-1)(x+1)^2(x^2+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)} + \frac{Dx+E}{(x^2+1)} \rightarrow$$

$$x + 1 = A(x + 1)^2(x^2 + 1) + B(x - 1)(x^2 + 1) + C(x - 1)(x + 1)(x^2 + 1) + (Dx + E)(x - 1)(x + 1)^2$$

$$x = -1; 0 = 4B; B = 0$$

$$x = 1; 2 = 8A; A = 4$$

$$\begin{cases} x = 0; & 1 = 4 - C - E \\ x = -2; & -1 = 20 - 9C + 6D - 3E \rightarrow C = -7; D = -9; E = 10 \\ x = 2; & 3 = 180 + 15C + 18D + 9E \end{cases}$$

$$\begin{aligned} \int \frac{x+1}{(x-1)(x+1)^2(x^2+1)} dx &= \int \frac{4}{(x-1)} dx + \int \frac{-7}{(x+1)} dx + \int \frac{-9x+10}{(x^2+1)} dx = \\ &= 4 \int \frac{1}{(x-1)} dx - 7 \int \frac{1}{(x+1)} dx + \int \frac{-9x}{(x^2+1)} dx + 10 \int \frac{1}{(x^2+1)} dx = \\ &= 4 \ln|x-1| - 7 \ln|x+1| - \frac{9}{2} \ln|x^2+1| + 10 \arctan x + C \end{aligned}$$

13. Halla las siguientes primitivas:

a) $\int \operatorname{sen}\left(\frac{3}{2}x + 1\right) dx; \quad t = \frac{3}{2}x + 1 \quad \frac{dt}{dx} = \frac{3}{2} \rightarrow dx = \frac{2 \cdot dt}{3}$

- Ahora sustituimos en la integral y la resolvemos.

$$\int \operatorname{sen}(t) \frac{2 \cdot dt}{3} \rightarrow \frac{2}{3} \cdot \int \operatorname{sen}(t) dt \rightarrow \frac{2}{3} \cdot (-\cos t) \rightarrow$$

- Sustituimos otra vez t por lo que teníamos y obtendremos el resultado.

$$\rightarrow -\frac{2 \cdot \cos\left(\frac{3}{2}x + 1\right)}{3} + C$$

b) $\int \frac{\operatorname{sen}(3x)}{\sqrt[3]{\cos(3x)}} dx = \int \frac{\operatorname{sen}(3x)}{(\cos(3x))^{\frac{1}{3}}} dx$

$$t = \cos(3x) \rightarrow \frac{dt}{dx} = -\sin(3x) \cdot 3 \rightarrow dx = \frac{dt}{-\sin(3x) \cdot 3}$$

Ahora lo sustituimos y realizamos la integral

$$\int \frac{\operatorname{sen}(3x)}{t^{\frac{1}{3}}} \cdot \frac{dt}{-\sin(3x) \cdot 3} \rightarrow \int -\frac{1}{t^{\frac{1}{3}}} \cdot \frac{1}{3} dt \rightarrow \int -\frac{1}{3t^{\frac{1}{3}}} dt \rightarrow -\frac{1}{3} \int \frac{1}{t^{\frac{1}{3}}} dt$$

$$\frac{1}{3} \int t^{-\frac{1}{3}} dt \rightarrow -\frac{1}{3} \cdot \frac{t^{\frac{2}{3}}}{\frac{2}{3}} \rightarrow -\frac{1}{3} \cdot \frac{3^{\frac{2}{3}} \sqrt[3]{\cos(3x)^2}}{2} \rightarrow -\frac{\sqrt[3]{\cos(3x)^2}}{2} + C$$

c) $\int \frac{\cot g(x)}{\operatorname{sen}^2(x)} dx = \int \frac{\frac{\cos(x)}{\operatorname{sen}(x)}}{\operatorname{sen}^2(x)} dx \rightarrow \int \frac{\cos(x)}{\operatorname{sen}^3(x)} dx$

$$t = \operatorname{sen}(x); \quad \frac{dt}{dx} = \cos(x) \rightarrow dx = \frac{dt}{\cos(x)};$$

$$\int \frac{\cos(x)}{t^3} \cdot \frac{dt}{\cos(x)} \rightarrow \int \frac{1}{t^3} dt \rightarrow \int t^{-3} dt \rightarrow \frac{t^{-2}}{-2} \rightarrow -\frac{1}{2\operatorname{sen}(x)^2} + C$$

$$\int \frac{\cot g(x)}{\operatorname{sen}^2(x)} dx = -\frac{1}{2\operatorname{sen}(x)^2} + C$$

$$\text{d)} \int \frac{\operatorname{sen} 2x}{(\cos 2x+1)^2} dx = -\frac{1}{2} \int \frac{(-2) \cdot \operatorname{sen} 2x}{(\cos 2x+1)^2} dx = -\frac{1}{2} \cdot \frac{-1}{(\cos 2x+1)} + C = \frac{1}{2(\cos 2x+1)} + C$$

$$t = (\cos 2x + 1) \quad dt = (-2) \cdot \operatorname{sen} 2x dx$$

$$\text{e)} \int (\tan x)^2 dx = \int (1 + (\tan x)^2 - 1) dx = \int (1 + (\tan x)^2) dx - \int 1 dx = \tan x - x + C$$

$$\text{f)} \int ((\tan x)^2 + x + 1) dx = \int (1 + (\tan x)^2 + x) dx = \int (1 + (\tan x)^2) dx + \int x dx =$$

$$= \tan x + \frac{x^2}{2} + C$$

$$\text{g)} \int \frac{\tan x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} (\tan x) dx = \frac{\tan^2 x}{2} + C$$

$$\text{h)} \int \frac{dx}{1-\operatorname{sen}^2 x} \quad \text{caso general} \quad \left[\begin{array}{l} t = tg \frac{x}{2} \quad \cos x = \frac{1-t^2}{1+t^2} \\ dx = \frac{2 \cdot dt}{1+t^2} \quad \operatorname{sen} x = \frac{2t}{1+t^2} \end{array} \right]$$

$$\int \frac{dx}{1-\operatorname{sen}^2 x} = \int \frac{\frac{2 \cdot dt}{1+t^2}}{1-\frac{4t^2}{1+t^2}} = \int \frac{\frac{2}{1+t^2}}{\frac{1+t^2-4t^2}{1+t^2}} dt = \int \frac{2}{(t-1)^2} dt = \frac{2 \cdot (t-1)^{-1}}{-1} + C = \frac{-2}{tg \frac{x}{2}-1} + C$$

$$\text{i)} \int \frac{dx}{\operatorname{sen} x} = \int \frac{1}{\operatorname{sen} x} dx \quad \text{Caso: } \frac{1}{-\operatorname{sen} x} = -\frac{1}{\operatorname{sen} x} \quad \text{Impar en seno}$$

$$\cos x = t; \quad \operatorname{sen} x = \sqrt{1 - \cos^2 x} = \sqrt{1 - t^2}$$

$$-\operatorname{sen} x dx = dt; \quad dx = \frac{dt}{-\operatorname{sen} x} = -\frac{dt}{\sqrt{1-t^2}}$$

$$\int \frac{dx}{\operatorname{sen} x} = \int \frac{1}{\operatorname{sen} x} dx = \int \frac{1}{\sqrt{1-t^2}} \cdot \frac{-dt}{\sqrt{1-t^2}} = \int \frac{1}{t^2-1} dt$$

$$t^2 - 1 = 0 \rightarrow t = \pm 1 \rightarrow \frac{1}{t^2-1} = \frac{A}{t-1} + \frac{B}{t+1}$$

$$1 = A(t+1) + B(t-1)$$

$$t = 1; \quad 1 = A2 \rightarrow A = \frac{1}{2}$$

$$t = -1; \quad 1 = -2B \rightarrow B = -\frac{1}{2}$$

$$\int \frac{1}{t^2-1} dt = \int \frac{\frac{1}{2}}{t-1} dt + \int \frac{-\frac{1}{2}}{t+1} dt \rightarrow \frac{1}{2} \ln|t-1| - \frac{1}{2} \ln|t+1| =$$

$$= \frac{1}{2} (\ln|\cos x - 1| - \ln|\cos x + 1|) + C = \frac{1}{2} \ln \frac{\cos x - 1}{\cos x + 1} + C$$

$$j) \int \sin^2 x \cos x \, dx ; \quad \sin x = t, \cos x \, dx = dt ;$$

$$\int \sin^2 x \cos x \, dx = \int t^2 dt = \frac{t^3}{3} + C = \frac{\sin^3 x}{3} + C$$

$$k) \int \sin^2 x \, dx ; \quad \sin^2 x = \frac{1 - \cos 2x}{2} ;$$

$$\int \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx = \frac{1}{2} \int (1 - \cos 2x) \, dx = \frac{1}{2} \int 1 \, dx - \frac{1}{2} \int \cos 2x \, dx =$$

$$\frac{1}{2} x - \frac{1}{2} \int \cos 2x \, dx = \frac{1}{2} x - \frac{1}{2} \cdot \frac{1}{2} \int \cos 2x \cdot 2 \, dx = \frac{1}{2} x - \frac{1}{4} \sin 2x + C = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$l) \int \sin^4 x \, dx ; \quad \sin^2 x = \frac{1 - \cos 2x}{2} ; \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\int \sin^4 x \, dx = \int (\sin^2 x)^2 \, dx = \int \left(\frac{1 - \cos 2x}{2} \right)^2 \, dx = \int \frac{(1 - \cos 2x)^2}{2^2} \, dx = \int \frac{1 - 2 \cos 2x + (\cos 2x)^2}{4} \, dx =$$

$$= \frac{1}{4} \int 1 \, dx - \frac{1}{4} \int 2 \cos 2x \, dx + \frac{1}{4} \int \frac{1 + \cos 4x}{2} \, dx = \frac{x}{4} - \frac{\sin 2x}{4} + \frac{1}{4} \cdot \frac{1}{2} \int 1 \, dx + \frac{1}{4} \cdot \frac{1}{2} \int \cos 4x \, dx =$$

$$\frac{x - \sin 2x}{4} + \frac{x}{8} + \frac{1}{8} \cdot \frac{1}{4} \int \cos 4x \cdot 4 \, dx = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$$

$$m) \int (\cos x)^4 \, dx = \int ((\cos x)^2)^2 \, dx = \frac{1}{2} \int (1 + \cos 2x)^2 \, dx =$$

$$= \frac{1}{2} \int 1 + 2 \cos 2x + (\cos 2x)^2 \, dx = \frac{1}{2} \int 1 \, dx + \frac{1}{2} \int 2 \cos 2x \, dx + \frac{1}{4} \int 1 + \cos 4x \, dx =$$

$$= \frac{1}{2} \int 1 \, dx + \frac{1}{2} \int 2 \cos 2x \, dx + \frac{1}{4} \int 1 \, dx + \frac{1}{8} \int 4 \cos 4x \, dx =$$

$$= \frac{1}{2} x + \frac{1}{2} \sin 2x + \frac{1}{4} x + \frac{1}{8} \sin 4x + C = \frac{1}{8} \sin 4x + \frac{1}{2} \sin 2x + \frac{3}{4} x + C$$

$$n) \int \cos(\ln x) \, dx$$

$$u = \cos(\ln x) \quad dv = dx$$

$$du = -\sin(\ln x) \frac{1}{x} \, dx \quad v = x$$

$$\int \cos(\ln x) \, dx = x \cdot \cos \ln x - \int x \left(-\sin(\ln x) \frac{1}{x} \right) dx = x \cos(\ln x) + \int \sin(\ln x) \, dx$$

$$\int u \, dv = u \, v - \int v \, du$$

$$u = \sin(\ln x) \quad dv = dx$$

$$du = \cos(\ln x) \frac{1}{x} \, dx \quad v = x$$

$$x \cos(\ln x) + x \sin(\ln x) - \int x \cos(\ln x) \frac{1}{x} \, dx =$$

$$= x \cos(\ln x) + x \sin(\ln x) - \int \cos(\ln x) \, dx$$

$$\int \cos(\ln x) dx = x \cos(\ln x) + x \operatorname{sen}(\ln x) - \int \cos(\ln x) dx$$

$$I = x \cos(\ln x) + x \operatorname{sen}(\ln x) - I$$

$$2I = x \cos(\ln x) + x \operatorname{sen}(\ln x); \quad I = \frac{x \cos(\ln x) + x \operatorname{sen}(\ln x)}{2}$$

$$\int \cos(\ln x) dx = \frac{x \cos(\ln x) + x \operatorname{sen}(\ln x)}{2} + C$$

$$\text{ñ)} \int \frac{(1+(\sin x)^2)}{\sin x \cos x} dx = \int \frac{1}{\sin x \cos x} dx + \int \frac{(\sin x)^2}{\sin x \cos x} dx$$

*Hacemos la primera integral:

$$\int \frac{1}{\sin x \cos x} dx$$

Sabemos que:

$$\left[\begin{aligned} \tan \frac{x}{2} = t \rightarrow dx &= \frac{2}{1+t^2} dt; \quad \sin x = \frac{2t}{1+t^2}; \quad \cos x = \frac{1-t^2}{1+t^2} \\ \sin x \cos x &= \frac{2t}{1+t^2} \cdot \frac{1-t^2}{1+t^2} = \frac{2t-2t^3}{(1+t^2)^2} \end{aligned} \right.$$

$$\int \frac{1}{\sin x \cos x} dx = \int \frac{1}{\frac{2t-2t^3}{(1+t^2)^2}} \cdot \frac{2}{1+t^2} dt = \int \frac{(1+t^2)^2}{2t-2t^3} \cdot \frac{2}{1+t^2} dt = \int \frac{2(1+t^2)}{2t+2t^3-2t^5} dt =$$

$$= \frac{1}{2} \cdot 2 \int \frac{(1+t^2)^2}{t-t^5} dt = \int \frac{(1+t^2)^2}{t(1-t^4)} dt = \int \frac{(1+t^2)^2}{t(1-t^2)(1+t^2)} dt = \int \frac{1+t^2}{t(1-t^2)} dt$$

$$\frac{1+t^2}{t(1-t^2)} = \frac{A}{t} + \frac{B}{1-t} + \frac{C}{1+t}; \quad 1+t^2 = A(1-t^2) + Bt(1+t) + Ct(1-t)$$

*Calculamos A, B y C dando valores a t:

$$\text{-Si } t=0; \quad 1 = A(1-0^2); \quad 1 = A$$

$$\text{-Si } t=1; \quad 2 = B \cdot 1 \cdot (1+1); \quad 1 = B$$

$$\text{-Si } t=-1; \quad 2 = 2C; \quad 1 = C$$

*Sustituimos los valores:

$$\int \left(\frac{1}{t} + \frac{1}{1-t} + \frac{1}{1+t} \right) dt = \ln|t| - \ln|1-t| + \ln|1+t|$$

*Sustituimos t por $\tan \frac{x}{2}$:

$$\int \frac{1}{\sin x \cos x} dx = \ln \left| \tan \frac{x}{2} \right| - \ln \left| 1 - \tan \frac{x}{2} \right| + \ln \left| 1 + \tan \frac{x}{2} \right|$$

*Realizamos la segunda integral:

$$\int \frac{\sin x}{\cos x} dx = -\ln|\cos x|$$

*Por tanto:

$$\int \frac{(1+(\sin x)^2)}{\sin x \cos x} dx = \ln \left| \tan \frac{x}{2} \right| - \ln \left| 1 - \tan \frac{x}{2} \right| + \ln \left| 1 + \tan \frac{x}{2} \right| - \ln |\cos x| + C =$$

$$\text{o) } \int \frac{dx}{1+\sin^2 x}$$

$$\begin{array}{lll} t = \operatorname{tg} x & \sin x = \frac{t}{\sqrt{t^2+1}} & \sin^2 x = \frac{t^2}{t^2+1} \\ dx = \frac{dt}{1+t^2} & \cos x = \frac{1}{\sqrt{t^2+1}} & \cos^2 x = \frac{1}{t^2+1} \end{array}$$

$$\begin{aligned} \int \frac{dx}{1+\sin^2 x} &= \int \frac{\frac{dt}{1+t^2}}{1+\frac{t^2}{t^2+1}} = \int \frac{\frac{dt}{1+t^2}}{\frac{t^2+1+t^2}{t^2+1}} = \int \frac{dt}{2t^2+1} = \int \frac{dt}{2\left(t^2+\frac{1}{2}\right)} = \frac{1}{2} \int \frac{dt}{t^2+\frac{1}{2}} = \\ &= \frac{1}{2} \cdot \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{t}{\frac{1}{\sqrt{2}}} + C = \frac{1}{2} \cdot \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\operatorname{tg} x}{\frac{1}{\sqrt{2}}} + C = \frac{1}{2} \cdot \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\operatorname{tg} x}{\frac{1}{\sqrt{2}}} + C = \frac{1}{\sqrt{2}} \operatorname{arctg}(\sqrt{2} \cdot \operatorname{tg} x) + C \end{aligned}$$

$$\text{p) } \int \sin 5x \cdot \cos 4x \, dx$$

$$\sin \alpha \cdot \cos \beta = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2}$$

$$\begin{aligned} \int \sin 5x \cdot \cos 4x \, dx &= \int \frac{\sin(5x+4x) + \sin(5x-4x)}{2} dx = \frac{1}{2} \int (\sin 9x + \sin x) dx = \\ &= \frac{1}{2} \left(\int \sin 9x \, dx + \int \sin x \, dx \right) = \frac{1}{2} \left(\frac{1}{9} \int \sin 9x \cdot 9 \, dx + \int \sin x \, dx \right) = \\ &= \frac{1}{2} \left(\frac{1}{9} \cdot (-\cos 9x) + (-\cos x) \right) + C = -\frac{1}{2} \left(\frac{\cos 9x}{9} + \cos x \right) + C \end{aligned}$$

$$\text{q) } \int \frac{dx}{13+12 \cos x}$$

$$\begin{array}{lll} t = \operatorname{tg} \frac{x}{2} & \frac{x}{2} = \operatorname{arctg} t \rightarrow x = 2 \operatorname{arctg} t & dx = \frac{2}{1+t^2} dt \\ \cos x = \frac{1-t^2}{1+t^2} & \sin x = \frac{2t}{1+t^2} & \operatorname{tg} x = \frac{2t}{1-t^2} \end{array}$$

$$\begin{aligned} \int \frac{dx}{13+12 \cos x} &= \int \frac{\frac{2dt}{1+t^2}}{13+12\left(\frac{1-t^2}{1+t^2}\right)} = \int \frac{\frac{2dt}{1+t^2}}{13+\frac{12-12t^2}{1+t^2}} = \int \frac{\frac{2dt}{1+t^2}}{\frac{13+13t^2+12-12t^2}{1+t^2}} = \int \frac{2dt}{t^2+25} = 2 \int \frac{1}{t^2+25} dt = \\ &= 2 \int \frac{1}{t^2+5^2} dt = 2 \cdot \frac{1}{5} \operatorname{arctg} \frac{t}{5} + C = \frac{2}{5} \operatorname{arctg} \left(\frac{\operatorname{tg} \frac{x}{2}}{5} \right) + C \end{aligned}$$

14. Resuelve las siguientes integrales definidas:

$$\text{a) } \int_0^6 (x^2 + x + 1) dx = \left. \frac{x^3}{3} + \frac{x^2}{2} + x \right|_0^6 = \left(\frac{6^3}{3} + \frac{6^2}{2} + 6 \right) - \left(\frac{0^3}{3} + \frac{0^2}{2} + 0 \right) = 92$$

$$\text{b) } \int_{-1}^1 (x^2 + x + 1) dx = \left. \frac{x^3}{3} + \frac{x^2}{2} + x \right|_{-1}^1 = \left(\frac{1^3}{3} + \frac{1^2}{2} + 1 \right) - \left(\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + (-1) \right) = \frac{8}{3}$$

$$c) \int_0^{\sqrt{3}} x \sqrt{x^2 + 1} dx = \frac{1}{2} \int_0^{\sqrt{3}} 2x(x^2 + 1)^{\frac{1}{2}} dx = \frac{1}{2} \frac{(x^2 + 1)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^{\sqrt{3}} = \frac{1}{3} \left[((\sqrt{3})^2 + 1)^{\frac{3}{2}} - (0^2 + 1)^{\frac{3}{2}} \right] = \frac{7}{3}$$

$$d) \int_{-1}^1 \frac{x+1}{x^2 + 2x + 2} dx = \frac{1}{2} \int_{-1}^1 \frac{2x+2}{x^2 + 2x + 2} dx = \frac{1}{2} \ln|x^2 + 2x + 2| \Big|_{-1}^1 = \frac{1}{2} [(\ln 5) - (\ln 1)] = \frac{\ln 5}{2}$$

$$e) \int_0^{\pi} \sin x dx = -\cos x \Big|_0^{\pi} = -\cos \pi + \cos 0 = 2$$

$$f) \int_1^e \ln x dx \quad \int \ln x dx = \begin{cases} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = dx \rightarrow v = \int dx = x \end{cases} = x \ln x - \int x \frac{1}{x} dx = x \ln x - x + C$$

$$\int_1^e \ln x dx = x \ln x - x \Big|_1^e = (e \ln e - e) - (1 \ln 1 - 1) = 1$$

15.

Halla el valor de c que verifica $\int_0^5 (2x+1) dx = f(c) \cdot (5-0)$ y razona su interpretación geométrica.

$$\int_0^5 (2x+1) dx = x^2 + x \Big|_0^5 = 30, \quad f(c) = 2c+1, \quad 30 = (2c+1) \cdot 5, \quad c = \frac{25}{10} = \frac{5}{2}$$

16. Sin efectuar el cálculo de la integral indefinida, calcula $f'(x)$ si $f(x) = \int_2^{e^x} \frac{dt}{\ln t}$

La función $g(t) = \frac{1}{\ln t}$ es continua en $[2, b]$, $g(x) = e^x$ es derivable,

Por el teorema fundamental del cálculo integral:

$$f'(x) = \frac{1}{\ln(e^x)} \cdot e^x$$

EJERCICIOS Y PROBLEMAS

1.

Sabiendo que $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ y $\int f^n(x) f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C$, calcula:

$$1) \int x^5 dx = \frac{x^6}{6} + C$$

$$2) \int \frac{4}{x^5} dx = \int 4x^{-5} dx = 4 \cdot \frac{x^{-4}}{-4} = \frac{-1}{x^4} + C$$

$$3) \int \frac{dx}{x^2} = \int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + C$$

$$4) \int 37 dx = 37x + C$$

$$5) \int 6x^7 dx = 6 \cdot \frac{x^8}{8} + C$$

$$6) \int 5x^{\frac{1}{4}} dx = 5 \cdot \frac{x^{\frac{5}{4}}}{\frac{5}{4}} = 4\sqrt[4]{x^5} + C$$

$$7) \int 5\sqrt{x^3} dx = \int 5x^{\frac{3}{2}} dx = 5 \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = 2\sqrt{x^5} + C$$

$$8) \int (3 - 2x - x^4) dx = 3x - \frac{2x^2}{2} - \frac{x^5}{5} + C = 3x - x^2 - \frac{x^5}{5} + C$$

$$9) \int (2x^5 - 5x + 3) dx = \frac{2x^6}{6} - \frac{5x^2}{2} + 3x + C$$

$$10) \int (2 + 3x^3)^2 dx = \int 4 + 9x^6 + 12x^3 dx = 4x + 9 \frac{x^7}{7} + 12 \frac{x^4}{4} + C = 4x + \frac{9x^7}{7} + 3x^4 + C$$

$$11) \int (2 \cdot (x^2 + 2)^3) dx = \int 2 \cdot ((x^2)^3 + (3x^2)^2 \cdot 2 + 3x^2 \cdot 2^2 + 2^3) dx = \int 2 \cdot (x^6 + 18x^4 + 12x^2 + 8) dx =$$

$$\int (2x^6 + 36x^4 + 24x^2 + 16) dx = 2 \cdot \frac{x^7}{7} + 36 \cdot \frac{x^5}{5} + 24 \cdot \frac{x^3}{3} + 16x + C$$

$$12) \int (1 - x^3)^2 dx = \int 1 - 2x^3 + (x^3)^2 dx = x - 2 \cdot \frac{x^4}{4} + \frac{x^7}{7} + C$$

$$13) \int \frac{x^3 - x + 2}{x^3} dx = \int \frac{x^3}{x^3} dx - \int \frac{x}{x^3} dx + \int \frac{2}{x^3} dx = \int 1 dx - \int \frac{1}{x^2} dx + \int \frac{2}{x^3} dx = \int (1 - x^{-2} + 2x^{-3}) dx =$$

$$= x + \frac{x^{-1}}{-1} + 2 \cdot \frac{x^{-2}}{-2} = x + \frac{x^{-1}}{1} - x^{-2} + C$$

$$14) \int (-4x^{\frac{2}{3}} + 2x) dx = -4 \cdot \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 2 \cdot \frac{x^2}{2} + C = -\frac{12x^{\frac{5}{3}}}{5} + x^2 + C$$

$$15) \int (3a \cdot \frac{1}{3e^2} + 2x^a) dx = (3a \cdot \frac{1}{3e^2})x + 2 \frac{x^{a+1}}{a+1} + C$$

$$16) \int -\frac{3}{x^3} + 2 - \frac{3}{\sqrt{x}} dx = \int -3x^{-3} + 2 - 3x^{-\frac{1}{2}} dx = -3 \frac{x^{-2}}{-2} + 2x - 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C =$$

$$= \frac{3}{2x^2} + 2x - 6\sqrt{x} + C$$

$$17) \int (3x^5 - \frac{4}{3x^2} + 2\sqrt[5]{x^2}) dx = \frac{3x^6}{6} - 12 \cdot \frac{x^{-1}}{-1} + 2 \cdot \frac{x^{\frac{7}{5}}}{\frac{7}{5}} = \frac{x^6}{2} - \frac{12}{x} + \frac{10x^{\frac{7}{5}}}{7} + C$$

$$18) \int (1-x)\sqrt{x} dx = \int \sqrt{x} - x\sqrt{x} dx = \int x^{\frac{1}{2}} - x \cdot x^{\frac{1}{2}} dx = \int x^{\frac{1}{2}} - x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2x^{\frac{3}{2}}}{3} - \frac{2x^{\frac{5}{2}}}{5} + C$$

$$19) \int \frac{x^3+5x^2-4}{x^2} dx = \int \frac{x^3}{x^2} dx + \int \frac{5x^2}{x^2} dx - \int \frac{4}{x^2} dx = \int x dx + \int 5 dx - \int 4x^{-2} dx =$$

$$= \frac{x^2}{2} + 5x - 4 \cdot \frac{x^{-1}}{-1} = \frac{x^2}{2} + 5x - \frac{4}{x} + C$$

$$20) \int (5e^x + \frac{2x^3-3x^2+5}{4x^2}) dx = \int 5e^x dx + \int \frac{2x^3}{4x^2} dx - \int \frac{3x^2}{4x^2} dx + \int \frac{5}{4x^2} dx =$$

$$= \int 5e^x dx + \int \frac{2x}{4} dx - \int \frac{3}{4} dx + \int \frac{5x^{-2}}{4} dx = 5e^x + \frac{1}{4}x^2 - \frac{3}{4}x - \frac{5x^{-1}}{4} =$$

$$= 5e^x + \frac{1}{4}x^2 - \frac{3}{4}x + \frac{5}{4x} + C$$

$$21) \int \frac{(1+x)^2}{\sqrt{x}} dx = \int (x^2 + 2x + 1) \left(x^{\frac{-1}{2}}\right) dx = \int (x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + x^{\frac{-1}{2}}) dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C =$$

$$\frac{2\sqrt{x^5}}{5} + \frac{4\sqrt{x^3}}{3} + 2\sqrt{x} + C$$

$$22) \int (\sqrt{x} - \frac{1}{2}x + \frac{2}{\sqrt{x}}) dx = \int (x^{\frac{1}{2}} - \frac{1}{2}x + 2x^{\frac{-1}{2}}) dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{4} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2\sqrt{x^3}}{3} - \frac{x^2}{4} + 4\sqrt{x} + C$$

$$23) \int \sqrt{x} (x^3 + 1) dx = \int (x^{\frac{1}{2}}) (x^3 + 1) dx = \int (x^{\frac{7}{2}}) + (x^{\frac{1}{2}}) dx = \frac{x^{\frac{9}{2}}}{\frac{9}{2}} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= \frac{2\sqrt{x^9}}{9} + \frac{2\sqrt{x^3}}{3} + C$$

$$24) \int (\sqrt{x^5} - \frac{2}{3\sqrt{x}}) dx = \int (x^{\frac{5}{2}} - \frac{2x^{\frac{-1}{2}}}{3}) dx = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} - \frac{2x^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} + C = \frac{2\sqrt{x^7}}{7} - \frac{4\sqrt{x}}{3} + C$$

$$25) \int \sqrt{x} (3 - 5x) dx = \int (x^{\frac{1}{2}}) (3 - 5x) dx = \int (3x^{\frac{1}{2}} - 5x^{\frac{3}{2}}) dx = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} + C =$$

$$= 2\sqrt{x^3} - 2\sqrt{x^5} + C$$

$$26) \int \frac{(x+1)+(x-2)}{\sqrt{x}} dx = \int (x^2 + x - 2) \left(x^{\frac{-1}{2}}\right) dx = \int (x^{\frac{3}{2}} + x^{\frac{1}{2}} - 2x^{\frac{-1}{2}}) dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} +$$

$$C = \frac{2\sqrt{x^5}}{5} + \frac{2\sqrt{x^3}}{3} - 4\sqrt{x} + C$$

$$27) \int (3x + 4)^2 dx = \int 3(3x + 4)^2 dx = \frac{(3x+4)^3}{3 \cdot 3} + C = \frac{(3x+4)^3}{9} + C$$

$$28) \int (3x - 7)^4 dx = \frac{1}{3} \int 3(3x - 7)^4 dx = \frac{(3x-7)^5}{3 \cdot 5} + C = \frac{(3x-7)^5}{15} + C$$

$$29) \int x (x^2 - 4)^3 dx = \frac{1}{2} \int 2x(x^2 - 4)^3 dx = \frac{(x^2-4)^4}{2 \cdot 4} + C = \frac{(x^2-4)^4}{8} + C$$

$$30. \int 3x (x^2 + 2)^3 dx = \frac{3}{2} \int 2x (x^2 + 2)^3 dx = \frac{3(x^2+2)^4}{2 \cdot 4} + C = \frac{3(x^2+2)^4}{8} + C$$

$$31. \int (x^3 + 2)^2 x^2 dx = \frac{1}{3} \int (x^3 + 2)^2 3x^2 dx = \frac{(x^3+2)^3}{3 \cdot 3} + C = \frac{(x^3+2)^3}{9} + C$$

$$32. \int (x^3 + 3) x^2 dx = \frac{1}{3} \int (x^3 + 3) 3x^2 dx = \frac{(x^3+3)^2}{3 \cdot 2} + C = \frac{(x^3+3)^2}{6} + C$$

$$33. \int (x - 2)^{3/2} dx = \frac{2\sqrt{(x-2)^5}}{5} + C$$

$$34. \int (a + x)^3 dx = \frac{(a+x)^4}{4} + C$$

$$35. \int [(x + 2)^3 - (x + 2)^2] dx = \frac{(x+2)^4}{4} - \frac{(x+2)^3}{3} + C$$

$$36. \int \sqrt{3x + 12} dx = \frac{1}{3} \int 3(3x + 12)^{\frac{1}{2}} dx = \frac{(3x+12)^{3/2}}{3 \cdot 3/2} + C = \frac{2\sqrt{(3x+12)^3}}{9} + C$$

$$37. \int \frac{dx}{\sqrt{x+3}} = \int (x+3)^{-\frac{1}{2}} dx = \frac{(x+3)^{1/2}}{1/2} + C = 2\sqrt{x+3} + C$$

$$38. \int \frac{dx}{(x-1)^3} = \int (x-1)^{-3} dx = \frac{(x-1)^{-2}}{-2} + C = \frac{-1}{2(x-1)^2} + C$$

$$39. \int (x^2 + x)^4 (2x + 1) dx = \frac{(x^2+x)^5}{5} + C$$

$$40. \int \frac{1}{\sqrt{x}} (1 + \sqrt{x})^2 dx = 2 \int \frac{1}{2\sqrt{x}} (1 + \sqrt{x})^2 dx = \frac{2(1+\sqrt{x})^3}{3} + C$$

$$41. \int \frac{x^3}{(x^4-1)^2} dx = \int x^3 (x^4 - 1)^{-2} dx = \frac{1}{4} \int 4x^3 (x^4 - 1)^{-2} dx = \frac{1}{4} \cdot \frac{(x^4-1)^{-1}}{-1} + c = \frac{(x^4-1)^{-1}}{-4} + c = \frac{1}{4(x^4-1)} + c$$

$$42. \int \frac{x}{(x^2+4)^3} dx = \int x(x^2 + 4)^{-3} dx = \frac{1}{2} \int 2x(x^2 + 4)^{-3} dx = \frac{1}{2} \cdot \frac{(x^2+4)^{-2}}{-2} + c = \frac{(x^2+4)^{-2}}{-4} + c = -\frac{1}{4(x^2+4)^2} + c$$

$$43. \int x\sqrt{x^2-7} dx = \int x(x^2-7)^{\frac{1}{2}} dx = \frac{1}{2} \int 2x(x^2-7)^{\frac{1}{2}} dx = \frac{1}{2} \cdot \frac{(x^2-7)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{(x^2-7)^{\frac{3}{2}}}{3} + c = \frac{\sqrt{(x^2-7)^3}}{3} + c$$

$$44. \int (x-1)(x^2-2x+3)^4 dx =$$

$$\frac{1}{2} \int (2x-2)(x^2-2x+3)^4 dx = \frac{1}{2} \cdot \frac{(x^2-2x+3)^5}{5} + c = \frac{(x^2-2x+3)^5}{10} + c$$

$$45. \int \frac{3x}{\sqrt{1+7x^2}} dx = \int 3x(1+7x^2)^{-\frac{1}{2}} dx =$$

$$\frac{3}{14} \int 14x(1+7x^2)^{-\frac{1}{2}} dx = \frac{3}{14} \cdot \frac{(1+7x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{3\sqrt{1+7x^2}}{7} + c$$

$$46. \int \frac{8x^2}{(x^3+2)^2} dx = \int 8x^2 (x^3 + 2)^{-2} dx =$$

$$\frac{8}{3} \int 3x^2 (x^3 + 2)^{-2} dx = \frac{8}{3} \cdot \frac{(x^3+2)^{-1}}{-1} + c = -\frac{8}{3(x^3+2)} + c$$

$$47. \int \frac{3x}{\sqrt[3]{x^2+3}} dx = \int 3x(x^2 + 3)^{-\frac{1}{3}} dx = \frac{3}{2} \int 2x(x^2 + 3)^{-\frac{1}{3}} dx =$$

$$\frac{3}{2} \cdot \frac{(x^2+3)^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{9 \sqrt[3]{(x^2+3)^2}}{4} + c$$

$$48. \int x \sqrt[3]{1-x^2} dx = \int x(1-x^2)^{\frac{1}{3}} dx = \frac{1}{2} \int 2x(1-x^2)^{\frac{1}{3}} dx =$$

$$\frac{1}{2} \cdot \frac{(1-x^2)^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3 \sqrt[3]{(1-x^2)^4}}{4} + c$$

$$49. \int \frac{x^2}{\sqrt[4]{x^3+5}} dx = \int x^2(x^3 + 5)^{-\frac{1}{4}} dx = \frac{1}{3} \int 3x^2(x^3 + 5)^{-\frac{1}{4}} dx = \frac{1}{3} \cdot \frac{(x^3+5)^{\frac{3}{4}}}{\frac{3}{4}} + c = \frac{4 \sqrt[4]{(x^3+5)^3}}{9} + c$$

$$50. \int x^2(x^3 - 1)^{\frac{3}{5}} dx = \frac{1}{3} \int 3x^2(x^3 - 1)^{\frac{3}{5}} dx =$$

$$\frac{1}{3} \cdot \frac{(x^3-1)^{\frac{8}{5}}}{\frac{8}{5}} + c = \frac{5 \sqrt[5]{(x^3-1)^8}}{24} + c$$

$$51. \int \sqrt{x^2 - 2x^4} dx = \int \sqrt{x^2(1 - 2x^2)} dx = \int x \sqrt{1 - 2x^2} dx = \int x(1 - 2x^2)^{\frac{1}{2}} dx = -\frac{1}{4} \int -4x(1 - 2x^2)^{\frac{1}{2}} dx =$$

$$-\frac{1}{4} \cdot \frac{(1-2x^2)^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{\sqrt{(1-2x^2)^3}}{6} + c$$

$$52. \int (e^x + 1)^3 e^x dx = \frac{(e^x+1)^4}{4} + c$$

$$53. \int \operatorname{sen}^3 x (\cos x) dx = \frac{\operatorname{sen}^4 x}{4} + c$$

$$54. \int x(\cos^4 x^2) \operatorname{sen} x^2 dx = -\frac{1}{2} \int 2x (\cos^4 x^2) (-\operatorname{sen} x^2) dx = -\frac{1}{2} \cdot \frac{\cos^5 x^2}{5} + c = -\frac{\cos^5 x^2}{10} + c$$

$$55. \int \frac{x \ln(x^2+3)}{x^2+3} dx = \int \ln(x^2 + 3) \cdot \frac{x}{x^2+3} dx =$$

$$\frac{1}{2} \int \ln(x^2 + 3) \cdot \frac{2x}{x^2+3} dx = \frac{1}{2} \cdot \frac{\ln^2|x^2+3|}{2} + c = \frac{\ln^2|x^2+3|}{4} + c$$

$$56. \int \frac{\operatorname{sen} x}{\cos^3 x} dx = \int \cos^{-3} x (\operatorname{sen} x) dx =$$

$$-1 \int \cos^{-3} x (-\operatorname{sen} x) dx = -1 \cdot \frac{\cos^{-2} x}{-2} + c = \frac{\cos^{-2} x}{2} + c = \frac{1}{2 \cos^2 x} + c$$

$$57. \int \frac{e^x}{2e^x-3} dx = \frac{1}{2} \int \frac{2e^x}{2e^x-3} dx = \frac{1}{2} \cdot \ln|2e^x - 3| + c$$

$$58. \int \operatorname{tg}^5 x (\sec^2 x) dx = \frac{\operatorname{tg}^6 x}{6} + c$$

$$59. \int \frac{\sec^2 3x}{\operatorname{tg} 3x} dx = \frac{1}{3} \int \frac{3 \cdot \sec^2 3x}{\operatorname{tg} 3x} dx = \frac{1}{3} \cdot \ln |\operatorname{tg} 3x| + c$$

$$60. \int \frac{\ln(x)}{3x} dx = \frac{1}{3} \int \ln |x| \cdot \frac{1}{x} dx = \frac{1}{3} \cdot \frac{\ln^2 |x|}{2} + c = \frac{\ln^2 |x|}{6} + c$$

2. Sabiendo que

$$\int \frac{1}{x} dx = \ln |x| + C \text{ y } \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C \text{ } \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C, \text{ calcula:}$$

$$1) \int \frac{dx}{x+2} = \int \frac{1}{x+2} dx = \ln |x+2| + C$$

$$2) \int \frac{dx}{2x-3} = \frac{1}{2} \ln |2x-3| + C$$

$$3) \int \frac{dx}{x-1} = \int \frac{1}{x-1} dx = \ln |x-1| + C$$

$$4) \int \frac{x dx}{x^2-1} = \frac{1}{2} \int \frac{2x}{x^2-1} dx = \frac{1}{2} \ln |x^2-1| + C$$

$$5) \int \frac{x}{1-2x^3} dx = \frac{-1}{6} \int \frac{-6x^2}{1-2x^3} = \frac{-1}{6} \ln |2x^3-1| + C$$

$$6) \int \frac{x^2}{1-x^3} dx = \frac{-1}{3} \int \frac{3x^2}{1-x^3} dx = \frac{-1}{3} \ln |x^3-1| + C$$

$$7) \int \frac{3x}{x^2+2} dx = \frac{3}{2} \int \frac{2x}{x^2+2} dx = \frac{3}{2} \ln |x^2+2| + C$$

$$8) \int \frac{4}{3x+5} dx = \frac{4}{3} \int \frac{3}{3x+5} dx = \frac{4}{3} \ln |3x+5| + C$$

$$9) \int \frac{x+1}{x^2+2x+2} dx = \frac{1}{2} \int \frac{2(x+1)}{x^2+2x+2} dx = \frac{1}{2} \int \frac{2x+2}{x^2+2x+2} dx = \frac{1}{2} \ln |x^2+2x+2| + C$$

$$10) \int \left(\sqrt{x} + \frac{1}{x} \right) dx = \int \sqrt{x} dx + \int \frac{1}{x} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \ln |x| + C$$

$$11) \int \left(\frac{3}{x^2} + \frac{2}{x} + \sqrt{x} \right) dx = \int 3x^{-2} + \frac{2}{x} + x^{\frac{1}{2}} dx = \frac{3x^{-1}}{-1} + 2\ln(x) + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = -3x^{-1} + 2\ln(x) + \frac{2\sqrt{x^3}}{3} + C$$

$$12) \int \frac{dx}{x \ln x} = \ln(|\ln |x||) + C$$

$$13) \int \frac{dx}{\sqrt{x}(1-\sqrt{x})} = \int \frac{1}{\sqrt{x}(1-\sqrt{x})} dx = -2 \int \frac{-1}{2\sqrt{x}(1-\sqrt{x})} dx = -2 \ln(|1-\sqrt{x}|) + C$$

$$14) \int \frac{1}{2x-1} - \frac{1}{2x+1} dx = \frac{1}{2} \left(\int \frac{2}{(2x-1)} dx - \int \frac{2}{(2x+1)} dx \right) = \frac{1}{2} \ln(|2x-1|) - \frac{1}{2} \ln(|2x+1|) + C$$

$$15) \int \frac{e^x}{e^x+1} dx = \ln(e^x + 1) + C$$

$$16) \int \frac{e^{2x}}{e^{2x}+3} dx = \frac{1}{2} \int \frac{2e^{2x}}{e^{2x}+3} dx = \frac{1}{2} \ln(e^{2x} + 3) + C$$

$$17) \int \tan(x) dx = \int \frac{\sin(x)}{\cos(x)} dx = -\ln(|\cos(x)|) + C$$

$$18) \int \frac{\cos(x)}{\sin(x)} dx = \ln(|\sin(x)|) + C$$

$$19) \int \frac{5}{x \ln(x)} dx = 5 \ln(|\ln(x)|) + C$$

$$20) \int \frac{\sin(x)+\cos(x)}{\cos(x)} dx = \int \left(\frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\cos(x)} \right) dx = \int \left(\frac{\sin(x)}{\cos(x)} + 1 \right) dx = \int \frac{\sin(x)}{\cos(x)} dx + \int 1 dx = -\ln(|\cos(x)|) + x + C$$

$$21) \int \frac{2 \sin(x) \cos(x)}{1+\sin(x^2)} dx = \ln(|1 + \sin(x^2)|) + C$$

$$22) \int \frac{\sin(x)-\cos(x)}{\sin(x)+\cos(x)} dx = -\ln(|\sin(x) + \cos(x)|) + C$$

$$23) \int x \cot x^2 dx = \frac{1}{2} \int 2x \frac{\cos(x^2)}{\sin(x^2)} dx = \frac{\ln(|\sin x^2|)}{2} + C$$

$$3. \text{ Si } \int e^x dx = e^x + C, \quad \int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + C,$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad y \quad \int a^{f(x)} \cdot f'(x) dx = \frac{a^{f(x)}}{\ln a} + C,$$

calcula:

$$1. \int 3^x dx = \frac{3^x}{\ln 3} + C$$

$$2. \int a^{4x} dx = \frac{1}{4} \int 4(a^{4x}) dx = \frac{a^{4x}}{4 \ln a} + C$$

$$3. \int e^{-x} dx = -1 \int (-1)(e^{-x}) dx = \frac{-1}{e^x} + C$$

$$4. \int 4e^{3x} dx = 4 \left(\frac{1}{3} \right) \int 3(e^{3x}) dx = \frac{4e^{3x}}{3} + C$$

$$5. \int (3x^2 \cdot e^{x^3+2}) dx = e^{x^3+2} + C$$

$$6. \int (4e^{4-x}) dx = 4 \cdot (-1) \int (-1)(e^{4-x}) dx = -4(e^{4-x}) + C$$

$$7. \int (x^2 e^{x^3}) dx = \frac{1}{3} \int [3x^2 (e^{x^3})] dx = \frac{e^{x^3}}{3} + C$$

$$8. \int (e^x + 1)^2 dx = \int [(e^x)^2 + 2(e^x)(1) + 1^2] dx = \int e^{2x} dx + \int 2e^x dx + \int 1 dx = \frac{1}{2} \int 2e^{2x} dx + \int 2e^x dx + \int 1 dx = \frac{e^{2x}}{2} + 2e^x + x + C$$

$$9. \int \left(e^x + \frac{1}{e^x} \right)^2 dx = \int (e^x + e^{-x})^2 dx = \int [(e^x)^2 + 2(e^x)(e^{-x}) + (e^{-x})^2] dx$$

$$= \int e^{2x} dx + \int 2dx + \int e^{-2x} dx = \frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + C$$

$$10. \int (e^x + x^6)^2 dx = \int [(e^x)^2 + 2(e^x)(x^6) + (x^6)^2] dx =$$

$$= \frac{1}{2} \int 2e^{2x} dx + \int 2(e^x)(x^6) dx \text{ (por partes)} + \int x^{12} dx =$$

$$= \frac{1}{2} e^{2x} + 2e^x (x^6 - 6x^5 + 30x^4 - 120x^3 + 360x^2 - 720x + 720) + \frac{x^{13}}{13} + C -$$

$$11. \int e^{-x^2+2} x dx = \frac{-1}{2} \int e^{-x^2+2} (-2x) dx = \frac{-(e^{-x^2+2})}{2} + C$$

$$12. \int \frac{e^{\ln x}}{x} dx = \int \frac{x}{x} dx = \int dx = x + C$$

$$13. \int \frac{e^{x^2}}{x^3} dx = \int e^{x^2} \cdot \frac{1}{x^3} dx = \frac{-1}{2} \int e^{x^2} \cdot \frac{-2}{x^3} dx = \frac{-1}{2} e^{x^2} + C$$

$$14. \int x e^{\sin x^2} \cos x^2 dx = \frac{1}{2} \int 2x e^{\sin x^2} \cos x^2 dx = \frac{e^{\sin x^2}}{2} + C$$

$$15. \int (e^{3 \cos 2x} \sin 2x) dx = \frac{-1}{6} \int (-6) (e^{3 \cos 2x} \sin 2x) dx = \frac{-e^{3 \cos 2x}}{6} + C$$

$$16. \int \frac{e^{\sqrt{x}}}{5\sqrt{x}} dx = \frac{1}{5} \cdot 2 \int \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx = \frac{2e^{\sqrt{x}}}{5} + C$$

$$17. \int e^{\cos x} \sin x dx = - \int e^{\cos x} (-\sin x) dx = -e^{\cos x} + C$$

$$18. \int \left(\frac{\sqrt{1+e^2}}{2e} - e^{x-3} \right) dx = \frac{\sqrt{1+e^2}}{2e} x - e^{x-3} + C$$

$$19. \int e^{\tan 2x} \sec^2 2x dx = \frac{1}{2} \int 2e^{\tan 2x} \sec^2 2x dx = \frac{e^{\tan 2x}}{2} + C$$

$$20. \int \frac{2x}{3} (3^{3+5x^2}) dx = \frac{2}{3} \int x (3^{3+5x^2}) dx = \frac{2}{3} \left(\frac{1}{10} \right) \int 10x (3^{3+5x^2}) dx = \frac{1}{15} \left(\frac{3^{3+5x^2}}{\ln 3} \right) + C$$

$$\int \frac{x}{2} (2^{3-5x^2}) dx = \frac{1}{2} \int x (2^{3-5x^2}) dx = \frac{1}{2} \cdot \left(-\frac{1}{10} \right) \int (-10x) (2^{3-5x^2}) dx = \frac{-1}{20} \left(\frac{2^{3-5x^2}}{\ln 2} \right) + C$$

4. Sabiendo que $\int \sin x dx = -\cos x + C$, $\int f'(x) \cdot \sin f(x) = -\cos f(x) + C$,

$\int \cos x dx = \sin x + C$ y $\int \cos f(x) \cdot f'(x) = \sin f(x) + C$ calcula:

$$1. \int \sin(2x + 8) dx = \frac{1}{2} \int \sin(2x + 8) 2 dx = \frac{-\cos(2x+8)}{2} + C$$

$$2. \int \sin \frac{x}{2} dx = 2 \int \sin \left(\frac{x}{2} \right) \left(\frac{1}{2} \right) dx = -2 \cos \frac{x}{2} + C$$

$$3. \int \cos 3x dx = \frac{1}{3} \int 3(\cos(3x)) dx = \frac{\sin 3x}{3} + C$$

$$4. \int x \sin x^2 dx = \frac{1}{2} \int 2x(\sin x^2) dx = \frac{-\cos x^2}{2} + C$$

$$5. \int \left(\frac{3 \sin x - 2 \cos x}{4} \right) dx = \frac{1}{4} \int (3 \sin x - 2 \cos x) dx = \frac{-3 \cos x}{4} - \frac{\sin x}{2} + C$$

$$6. \int \sin 2x dx = \frac{1}{2} \int 2(\sin 2x) dx = \frac{-\cos 2x}{2} + C$$

7. $\int e^x \cos e^x dx = \sin e^x + C$
8. $\int x \cos(2x^2) \cdot \sin(2x^2) dx = \frac{1}{4} \int 4x \cos(2x^2) \cdot \sin(2x^2) dx = -\frac{1}{4} \cos(2x^2) + C$
9. $\int \frac{\sin(\ln x)}{x} dx = \int \frac{1}{x} \cdot \sin(\ln x) dx = -\cos(\ln x) + C$

7. Si

$$\int \frac{1}{\cos^2 x} dx = \int (1 + \tan^2 x) dx = \tan x + C \quad \text{y} \quad \int \frac{f'(x)}{\cos^2 f(x)} dx = \int [1 + \tan^2 f(x)] \cdot f'(x) dx = \tan f(x) + C, \text{ calcula:}$$

- 1) $\int x(1 + \tan x^2) dx = \frac{1}{2} \int 2x(1 + \tan x^2) dx = \frac{1}{2} \tan(x^2) + C$
- 2) $\int (1 + \tan x)^2 dx = \int (1 + \tan^2 x + 2 \tan x) dx = \int ((1 + \tan^2 x) + 2 \tan x) dx = \tan x - 2 \int \frac{-\sin x}{\cos x} dx = \tan x - 2 \ln|\cos x| + C$
- 3) $\int \tan^2 3x dx = \int (1 - 1 + \tan^2 3x) dx = \frac{1}{3} \int 3(1 + \tan^2 3x) dx - \int 1 dx = \frac{1}{3} \cdot \tan 3x - x + C$

6. Halla el valor de las siguientes integrales, usando un cambio de variable:

1) $\int (2 + 5x)^4 dx = \int t^4 \cdot \frac{1}{5} dt = \frac{1}{5} \int t^4 dt = \frac{1}{5} \cdot \frac{t^5}{5} = \frac{(2+5x)^5}{25} + c$

$$t = 2+5x, \quad dt = 5 dx \rightarrow dx = \frac{1}{5} dt$$

2) $\int (3 + 4x)^6 dx = \int t^6 \cdot \frac{1}{4} dt = \frac{1}{4} \int t^6 dt = \frac{1}{4} \cdot \frac{t^7}{7} = \frac{t^7}{28} = \frac{(3+4x)^7}{28} + c$

$$t = 3+4x, \quad dt = 4x dx \rightarrow dx = \frac{1}{4} dt$$

3) $\int 6x(3 + x^2)^5 dx = \int 6x \cdot t^5 \cdot \frac{1}{2x} dt = \int 3t^5 dt = 3 \int t^5 dt = \frac{3t^6}{6} = \frac{t^6}{2} = \frac{(3+x^2)^6}{2} + c$

$$t = 3 + x^2, \quad dt = 2x dx \rightarrow dx = \frac{1}{2x} dt$$

4) $\int \left[\frac{3}{5+4x} + \frac{3}{(5+4x)^3} \right] dx = \int \left[\frac{3}{t} \cdot \frac{1}{4} \right] dt + \int \left[\frac{3}{t^3} \cdot \frac{1}{4} \right] dt = \int \left[\frac{3}{4t} \right] dt + \int \left[\frac{3}{4t^3} \right] dt =$

$$\frac{3}{4} \int \left[\frac{1}{t} \right] dt + \frac{3}{4} \int \left[\frac{1}{t^3} \right] dt = \frac{3}{4} \int \left[\frac{1}{t} \right] dt + \frac{3}{4} \int [t^{-3}] dt = \frac{3}{4} \ln|t| + \frac{3}{4} \cdot \frac{t^{-2}}{-2} = \frac{3}{4} \ln|t| - \frac{3t^{-2}}{8} =$$

$$\frac{3}{4} \ln|t| - \frac{3}{8t^2} = \frac{3}{4} \ln|5 + 4x| - \frac{3}{8(5+4x)^2} + c$$

$$t = 5+4x \quad , \quad dt = 4 dx \rightarrow dx = \frac{1}{4} dt$$

$$\begin{aligned} 5) \quad \int (\sqrt{3+2x} + \sqrt[3]{3+2x}) dx &= \int (\sqrt{t} + \sqrt[3]{t}) \cdot \frac{1}{2} dt = \frac{1}{2} \int (t^{\frac{1}{2}} + t^{\frac{1}{3}}) dt = \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} \cdot \frac{t^{\frac{4}{3}}}{\frac{4}{3}} = \\ &= \frac{t^{\frac{3}{2}}}{3} + \frac{3t^{\frac{4}{3}}}{8} = \frac{\sqrt{t^3}}{3} + \frac{3\sqrt[3]{t^4}}{8} = \frac{\sqrt{(3+2x)^3}}{3} + \frac{3\sqrt[3]{(3+2x)^4}}{8} + c \end{aligned}$$

$$t = 3+2x \quad , \quad dt = 2 dx \rightarrow dx = \frac{1}{2} dt$$

$$\begin{aligned} 6) \quad \int \left(\frac{e^x-4}{e^{2x}}\right) dx &= \int \left(\frac{t-4}{t^2}\right) \cdot \frac{1}{t} dt = \int \left(\frac{t-4}{t^3}\right) dt = \int \left(\frac{t}{t^3}\right) dt - \int \left(\frac{4}{t^3}\right) dt = \\ &= \int \left(\frac{1}{t^2}\right) dt - 4 \int \left(\frac{1}{t^3}\right) dt = \int t^{-2} dt - 4 \int t^{-3} dt = \frac{t^{-1}}{-1} - \frac{4t^{-2}}{-2} = \frac{-1}{t} + \frac{2}{t^2} = \frac{-1}{e^x} + \frac{2}{e^{2x}} + c \end{aligned}$$

$$t = e^x \quad , \quad dt = e^x dx \rightarrow dx = \frac{1}{e^x} dt = \frac{1}{t} dt$$

$$7) \quad \int \sin^3(x) \cdot \cos(x) dx = \int t^3 \cdot \cos(x) \cdot \frac{1}{\cos(x)} dt = \int t^3 dt = \frac{t^4}{4} = \frac{\sin^4(x)}{4} + c$$

$$t = \sin(x) \quad , \quad dt = \cos(x) dx \rightarrow dx = \frac{1}{\cos(x)} dt$$

$$8) \quad \int \left(\frac{\sin(x)}{\cos(x)}\right) dx = \int \left(\frac{\sin(x)}{t}\right) \cdot \frac{-1}{\sin(x)} dt = \int \frac{-1}{t} dt = -\ln|t| = -\ln|\cos(x)| + c$$

$$t = \cos(x) \quad , \quad dt = -\sin(x) dx \rightarrow dx = \frac{-1}{\sin(x)} dt$$

$$9) \quad \int \left(\frac{\cos(x)}{\sin^4(x)}\right) dx = \int \left(\frac{\cos(x)}{t^4}\right) \cdot \frac{1}{\cos(x)} dt = \int \left(\frac{1}{t^4}\right) dt = \int t^{-4} dt = \frac{t^{-3}}{-3} = \frac{-1}{3t^3} = \frac{-1}{3\sin^3(x)} + c$$

$$t = \sin(x) \quad , \quad dt = \cos(x) dx \rightarrow dx = \frac{1}{\cos(x)} dt$$

$$10) \quad \int x\sqrt{x^2+4} dx = \int x\sqrt{t} \cdot \frac{1}{2x} dt = \int \left(\frac{\sqrt{t}}{2}\right) dt = \int \left(\frac{t^{\frac{1}{2}}}{2}\right) dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2} \cdot 2} = \frac{2t^{\frac{3}{2}}}{6} = \frac{t^{\frac{3}{2}}}{3} = \frac{\sqrt{(x^2+4)^3}}{3} + c$$

$$t = x^2 + 4 \quad , \quad dt = 2x dx \rightarrow dx = \frac{1}{2x} dt$$

$$11) \quad \int \left(\frac{e^x+3}{e^{2x}}\right) dx = \int \left(\frac{t+3}{t^2}\right) \cdot \frac{1}{t} dt = \int \left(\frac{t+3}{t^3}\right) dt = \int \left(\frac{t}{t^3}\right) dt + \int \left(\frac{3}{t^3}\right) dt =$$

$$\int t \cdot t^{-3} dt + \int 3t^{-3} dt = \int t^{-2} dt + \int 3t^{-3} dt = \frac{t^{-1}}{-1} + \frac{3t^{-2}}{-2} = \frac{-1}{t} - \frac{3}{2t^2} = \frac{-1}{e^x} - \frac{3}{2e^{2x}} + c$$

$$t = e^x, \quad dt = e^x dx \rightarrow dx = \frac{1}{e^x} dt = \frac{1}{t} dt$$

$$\begin{aligned} 12) \int \left(\frac{e^{-x} + 2}{e^{3x}} \right) dx &= \int \left(\frac{\frac{1}{t} + 2}{t^3} \cdot \frac{1}{t} \right) dt = \int \left(\frac{1 + 2t}{t^5} \right) dt = \int \left(\frac{1}{t^5} \right) dt + \int \left(\frac{2t}{t^5} \right) dt = \\ &= \int t^{-5} dt + \int 2t^{-4} dt = \frac{t^{-4}}{-4} + \frac{2t^{-3}}{-3} = \frac{e^{-4x}}{-4} - \frac{2e^{-3x}}{3} + C \end{aligned}$$

$$t = e^x, \quad dt = e^x dx \rightarrow dx = \frac{1}{e^x} dt = \frac{1}{t} dt; \quad e^{-x} = t^{-1} = \frac{1}{t}$$

$$13) \int \frac{(x - 2\sqrt{x})^2}{3x^2} dx$$

$$\begin{aligned} \int \frac{(x - 2\sqrt{x})^2}{3x^2} dx &= \frac{1}{3} \int \left(\frac{x^2 - 4\sqrt{x} + 4x}{x^2} \right) dx = \frac{1}{3} \int \left(\frac{x^2}{x^2} - \frac{4\sqrt{x}}{x^2} + \frac{4x}{x^2} \right) dx = \frac{1}{3} \left(\int 1 dx - 4 \int x^{-\frac{3}{2}} dx + 4 \int \frac{1}{x} dx \right) = \\ &= \frac{1}{3} \left(x - 4 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 4 \ln|x| \right) + C = \frac{1}{3} \left(x + \frac{8}{\sqrt{x}} + 4 \ln|x| \right) + C \end{aligned}$$

$$14) \int \frac{(2 + 3\sqrt{x})^2}{4x} dx$$

$$\begin{aligned} \int \frac{(2 + 3\sqrt{x})^2}{4x} dx &= \frac{1}{4} \int \left(\frac{2^2 + 12\sqrt{x} + 9x}{x} \right) dx = \frac{1}{4} \int \left(\frac{4}{x} - \frac{12\sqrt{x}}{x} + \frac{9x}{x} \right) dx = \frac{1}{4} \left(\int \frac{4}{x} dx + 12 \int x^{-\frac{1}{2}} dx + \int 9 dx \right) = \\ &= \frac{1}{4} \left(4 \ln|x| + 12 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 9x \right) + C = \ln|x| + 6\sqrt{x} + \frac{9x}{4} + C \end{aligned}$$

$$15) \int \operatorname{sen} \sqrt{x} dx; \quad x = t^2; \quad dx = 2t dt; \quad t = \sqrt{x}$$

$$\int \operatorname{sen} \sqrt{t^2} 2t dt = \int \operatorname{sen} t 2t dt \quad u = 2t, \quad du = 2dt; \quad dv = \operatorname{sen} t dt, \quad v = -\cos t$$

$$\int \operatorname{sen} t 2t dt = -2t \cos t + \int \cos t 2t dt = -2t \cos t + 2 \operatorname{sen} t + C = 2\sqrt{x} \cos \sqrt{x} + 2 \operatorname{sen} \sqrt{x} + C$$

7. Halla el valor de las siguientes integrales, usando el método de integración por partes.

$$\begin{aligned} 1) \quad \int 3x \cos x dx &= \quad (dv = \cos x dx \quad u = 3x; \quad v = \operatorname{sen} x \quad du = 3 dx) \\ &= 3x \cdot (\operatorname{sen} x) - \int \operatorname{sen} x \cdot 3 dx = 3x \cdot \operatorname{sen} x + 3 \cdot \cos x + C \end{aligned}$$

$$\begin{aligned} 2) \quad \int x^2 \cdot \operatorname{sen} x dx &= \quad (dv = \operatorname{sen} x dx \quad u = x^2; \quad v = -\cos x \quad du = 2x dx) \\ &= x^2 \cdot (-\cos x) - \int (-\cos x) \cdot 2x dx = \quad (dv = -\cos x dx \quad u = 2x; \quad v = -\operatorname{sen} x \quad du = 2 dx) \\ &= (x^2 \cdot (-\cos x)) - [2x \cdot (-\operatorname{sen} x) - \int (-\operatorname{sen} x) \cdot 2 dx] = \end{aligned}$$

$$= x^2 \cdot (-\cos x) + 2x \cdot (\sin x) - 2 \cdot (-\cos x) + C =$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

3) $\int x^2 \ln x \, dx =$ ($dv = x^2 \, dx$ $u = \ln x$; $v = \frac{x^3}{3}$ $du = \frac{1}{x}$)

$$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx = \ln x \cdot \frac{x^3}{3} - \int \frac{x^2}{3} \, dx =$$

$$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 \, dx = \ln x \cdot \frac{x^3}{3} - \frac{x^3}{9} + C$$

4) $\int \sqrt{x} \cdot \ln x \, dx =$ ($dv = \sqrt{x} \, dx = x^{\frac{1}{2}} \, dx$ $u = \ln x$; $v = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ $du = \frac{1}{x} \, dx$)

$$= \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \cdot \frac{1}{x} \, dx = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{2x^{\frac{1}{2}}}{3} \, dx =$$

$$= \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2}{3} \int x^{\frac{1}{2}} \, dx = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4x^{\frac{3}{2}}}{9} + C$$

5) $\int \frac{\ln x}{x^2} \, dx =$ ($dv = x^{-2} \, dx$ $u = \ln x$; $v = \frac{x^{-1}}{-1}$ $du = \frac{1}{x} \, dx$)

$$= \int \ln x \cdot x^{-2} \, dx = \ln x \cdot \frac{x^{-1}}{-1} - \int \frac{x^{-1}}{-1} \cdot \frac{1}{x} \, dx = \ln x \cdot \frac{x^{-1}}{-1} - \int \frac{x^{-2}}{-1} \, dx =$$

$$= \ln x \cdot \frac{x^{-1}}{-1} + \int \frac{1}{x^2} \, dx = -\ln x \cdot \frac{1}{x} + \int \frac{1}{x^2} \, dx = -\ln x \cdot \frac{1}{x} + \frac{x^{-1}}{-1} + C =$$

$$= -\ln x \cdot \frac{1}{x} - \frac{1}{x} + C$$

6) $\int 2e^x \cdot \cos x \, dx =$

Cambios 1	Cambios 2
$dv = \cos x \, dx$ $v = \sin x$	$dv = \sin x$ $v = -\cos x$
$u = 2e^x$ $du = 2e^x \, dx$	$u = 2e^x$ $du = 2e^x \, dx$

(cambios 1) $= 2e^x \cdot \sin x - \int \sin x \cdot 2e^x \, dx =$

(cambios 2) $= 2e^x \cdot \sin x - [2e^x \cdot (-\cos x) - \int (-\cos x) \cdot 2e^x \, dx] =$

$$= 2e^x \cdot \sin x - [2e^x \cdot (-\cos x) + \int \cos x \cdot 2e^x \, dx] =$$

$$= 2e^x \cdot \sin x + 2e^x \cdot \cos x - \int \cos x \cdot 2e^x \, dx \quad (\text{haciendo } \int \cos x \cdot 2e^x \, dx = I)$$

$$I = 2e^x \cdot \sin x + 2e^x \cdot \cos x - I$$

$$2I = 2e^x \cdot \sin x + 2e^x \cdot \cos x$$

$$I = e^x \cdot \sin x - e^x \cdot \cos x + C$$

7) $\int 2e^x \cdot \sen x \, dx$

- Fórmula: $\int u \cdot dv = u \cdot v - \int v \cdot du$

- $u = 2e^x \rightarrow du = 2e^x \, dx$

$$- dv = \operatorname{sen} x dx \rightarrow v = \int \operatorname{sen} x dx = -\cos x$$

$$\int 2e^x \cdot \operatorname{sen} x dx = 2e^x \cdot (-\cos x) - \int -\cos x \cdot 2e^x dx = 2e^x \cdot (-\cos x) + \int \cos x \cdot 2e^x dx =$$

Repetimos el método de integración por partes:

$$- u = 2e^x \rightarrow du = 2e^x dx$$

$$- dv = \cos x dx \rightarrow v = \int \cos x dx = \operatorname{sen} x$$

$$= 2e^x \cdot (-\cos x) + 2e^x \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot 2e^x dx$$

Hacemos $\int \operatorname{sen} x \cdot 2e^x = I$ de donde,

$$I = 2e^x \cdot (-\cos x) + 2e^x \cdot \operatorname{sen} x - I$$

$$2I = 2e^x \cdot (-\cos x) + 2e^x \cdot \operatorname{sen} x$$

$$I = e^x \cdot (-\cos x) + e^x \cdot \operatorname{sen} x$$

$$I = e^x \cdot (\operatorname{sen} x - \cos x) + C$$

8) $\int e^x \cdot \cos 3x dx$

$$- u = e^x \rightarrow du = e^x dx$$

$$- dv = \cos 3x dx \rightarrow v = \int \cos 3x dx = \frac{1}{3} \cdot \operatorname{sen} 3x$$

$$\int e^x \cdot \cos 3x dx = \frac{e^x}{3} \operatorname{sen} 3x - \int \frac{e^x}{3} \operatorname{sen} 3x dx = \frac{e^x}{3} \operatorname{sen} 3x - \frac{1}{3} \int e^x \operatorname{sen} 3x dx =$$

Repetimos el método de integración por partes:

$$- u = e^x \rightarrow du = e^x dx$$

$$- dv = \operatorname{sen} 3x dx \rightarrow v = \int \operatorname{sen} 3x dx = -\frac{1}{3} \cos 3x$$

$$= \frac{e^x}{3} \operatorname{sen} 3x - \frac{1}{3} \left(-\frac{e^x}{3} \cdot \cos 3x - \frac{1}{3} \cdot \int e^x (-\cos 3x) dx \right)$$

Hacemos $\int e^x \cos 3x dx = I$, de donde,

$$I = \frac{e^x}{3} \operatorname{sen} 3x + \frac{e^x}{9} \cos 3x - \frac{1}{9} \cdot I$$

$$\frac{10}{9} \cdot I = \frac{e^x}{3} \cdot \operatorname{sen} 3x + \frac{e^x}{9} \cos 3x$$

$$I = \frac{e^x}{10} \cdot \left(3 \operatorname{sen} 3x + \frac{\cos 3x}{3} \right) + C$$

9) $\int \frac{4-2x^2}{x} \cdot \ln(x) dx$

$$- u = \ln(x) \rightarrow du = \frac{1}{x} dx$$

$$- dv = \frac{4-2x^2}{x} dx \rightarrow v = \int \left(\frac{4}{x} - 2x \right) dx = 4 \cdot \ln(x) - x^2$$

$$\int \frac{4-2x^2}{x} \cdot \ln(x) dx = \ln(x) \cdot (4 \cdot \ln(x) - x^2) - \int \frac{1}{x} \cdot (4 \ln(x) - x^2) dx =$$

$$\begin{aligned}
&= \ln(x) \cdot (4 \cdot \ln(x) - x^2) - \int \frac{4}{x} \cdot \ln(x) dx + \int x dx = \\
&= \ln(x) \cdot (4 \cdot \ln(x) - x^2) + \frac{x^2}{2} - 4 \int \frac{\ln(x)}{x} dx = \left[4 \int \ln(x) \cdot \frac{1}{x} dx = 4 \frac{\ln^2(x)}{2} \right] \\
&= 4 \ln^2(x) - x^2 \ln(x) + \frac{x^2}{2} - \frac{4 \ln^2(x)}{2} = 2 \ln^2(x) - x^2 \ln(x) + \frac{x^2}{2} + C
\end{aligned}$$

8. Halla el valor de las siguientes integrales racionales:

1) $\int \frac{2}{x^2+1} dx$

Las raíces del denominador son raíces complejas.

$$x^2 + 1 = 0; \quad x = \pm i \text{ luego:}$$

$$\int \frac{2}{x^2+1} dx = 2 \int \frac{1}{x^2+1} dx = 2 \arctan(x) + C$$

2) $\int \frac{3}{2x^2+2} dx$

Las raíces del denominador son raíces complejas.

$$2x^2 + 2 = 0; \quad x = \pm i \text{ luego:}$$

$$\int \frac{3}{2x^2+2} dx = \int \frac{3}{2(x^2+1)} dx = \frac{3}{2} \int \frac{1}{x^2+1} dx = \frac{3}{2} \arctan(x) + C = \frac{3 \arctan(x)}{2} + C$$

3) $\int \frac{3}{x-3} dx$

La raíz del denominador es una raíz real simple.

$$x - 3 = 0; \quad x = 3 \text{ luego:}$$

$$\int \frac{3}{x-3} dx = 3 \int \frac{1}{x-3} dx = 3 \ln|x-3| + C$$

4) $\int \frac{2}{3x^2+3} dx$

$$\int \frac{2}{3x^2+3} dx = \int \frac{2}{3(x^2+1)} dx = \frac{2}{3} \int \frac{1}{x^2+1} dx = \frac{2}{3} \arctan(x) + C = \frac{2 \arctan(x)}{3} + C$$

5) $\int \frac{5x}{x^2+3} dx$

$$\int \frac{5x}{x^2+3} dx = \frac{5}{2} \int \frac{2x}{(x^2+3)} dx = \frac{5}{2} \ln|x^2+3| + C$$

6) $\int \frac{3x-2}{x^2+1} dx$

$$\int \frac{3x-2}{x^2+1} dx = \int \left(\frac{3x}{x^2+1} - \frac{2}{x^2+1} \right) dx = \frac{3}{2} \int \frac{2x}{x^2+1} dx - 2 \int \frac{1}{x^2+1} dx = \frac{3 \ln|x^2+1|}{2} - 2 \arctan(x) + C$$

$$7. \int \frac{(2x-3)^2}{3x^2} dx$$

$$\begin{aligned} \int \frac{(2x-3)^2}{3x^2} dx &= \int \frac{4x^2+9-12x}{3x^2} dx = \int \frac{4}{3} dx + \int \frac{-12x}{3x^2} dx + \int \frac{9}{3x^2} dx = \\ &= \frac{4}{3} \int dx - 4 \int \frac{1}{x} dx + 3 \int \frac{1}{x^2} dx = \frac{4}{3}x - 4 \ln|x| + 3 \int x^{-2} dx = \\ &= \frac{4}{3}x - 4 \ln|x| + 3 \frac{x^{-1}}{-1} + C = \frac{4}{3}x - 4 \ln|x| - \frac{3}{x} + C \end{aligned}$$

$$8. \int \frac{x+2}{x+1} dx$$

Según la división euclídea, obtenemos:

$$C(x)=1 \quad R(x)=1 \quad Q(x)=x+1$$

$$\int \frac{x+2}{x+1} dx = \int \left(1 + \frac{1}{x+1}\right) dx = \int 1 dx + \int \frac{1}{x+1} dx = x + \ln|x+1| + C$$

$$9. \int \frac{x-1}{x+1} dx$$

Según la división euclídea, obtenemos:

$$C(x)=1 \quad R(x)=-2 \quad Q(x)=x+1$$

$$\int \frac{x-1}{x+1} dx = \int \left(1 + \frac{-2}{x+1}\right) dx = \int 1 dx + \int \frac{-2}{x+1} dx = x - 2 \ln|x+1| + C$$

$$10) \int \frac{3x-1}{x+3} dx;$$

Efectuando la división obtenemos:

$$C(x): 3$$

$$R(x): -10$$

$$Q(x): x+3$$

$$\int \frac{P(x)}{Q(x)} dx = \int C(x) dx + \int \frac{R(x)}{Q(x)} dx$$

$$\int \frac{3x-1}{x+3} dx = \int 3 dx + (-10) \int \frac{1}{x+3} dx; 3x - 10 \ln(|x+3|) + C$$

$$11) \int \frac{3x^3}{x^2-4} dx; \quad C(x): 3x \quad R(x): 12x \quad Q(x): x^2-4$$

$$\int \frac{3x^3}{x^2-4} dx = \int 3x dx + 6 \int \frac{12x}{x^2-4} dx; \frac{3x^2}{2} + 6 \ln(|x^2-4|) + C$$

$$12) \int \frac{3x^3}{x^2-1} dx; \quad C(x): 3x \quad R(x): 3x \quad Q(x): x^2-1$$

$$\int \frac{3x^3}{x^2-1} dx = \int 3x dx + \int \frac{3x}{x^2-1} dx; \int 3x dx + \frac{3}{2} \int \frac{2x}{(x^2-1)} dx; \frac{3x^2}{2} + \frac{3}{2} \ln(|x^2-1|) + C$$

13) $\int \frac{x^2+2x+2}{x+2} dx$; efectuando la división obtenemos $C(x) = x$; $R(x) = 2$; de donde

$$\int \frac{x^2+2x+2}{x+2} dx = \int (x) dx + \int \frac{2}{x+2} dx = \int (x) dx + 2 \int \frac{1}{x+2} dx = \frac{x^2}{2} + 2 \ln|x+2| + C$$

14) $\int \frac{x^3+4x^2-2x+5}{x-2} dx$; efectuando la división, $C(x) = x^2 + 6x + 10$; $R(x) = -25$; de donde

$$\int \frac{x^3+4x^2-2x+5}{x-2} dx = \int (x^2 + 6x + 10) dx + \int \frac{-25}{x-2} dx =$$

$$= \int (x^2 + 6x + 10) dx - 25 \int \frac{1}{x-2} dx = \frac{x^3}{3} + 3x^2 + 10x - 25 \ln|x-2| + C$$

15) $\int \frac{2}{x^2-4} dx$; hacemos la descomposición en fracciones simples,

$$\frac{2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} \quad 2 = A(x+2) + B(x-2), \text{ sustituyendo por } x = -2, A = \frac{1}{2}, B = -\frac{1}{2}$$

$$\int \frac{2}{(x-2)(x+2)} dx = \int \frac{A}{x-2} dx + \int \frac{B}{x+2} dx = \int \frac{1}{2(x-2)} dx + \int -\frac{1}{2(x+2)} dx = \int \frac{1}{2x-2} dx - \int \frac{1}{2x+2} dx =$$

$$\frac{1}{2} \int \frac{2}{2x-2} dx - \frac{1}{2} \int \frac{2}{2x+2} dx = \frac{1}{2} \ln|x-2| - \frac{1}{2} \ln|x+2| + C = \frac{1}{2} \ln \left| \frac{x-2}{x+2} \right| + C$$

16) $\int \frac{3x+2}{x^2+3x} dx$

1. Se analizan las raíces del denominador: $x^2 + 3x = 0$; $x(x+3) = 0 \rightarrow x = 0$; $x = -3$

2. Se escribe la descomposición en fracciones simples:

$$\frac{3x+2}{x^2+3x} = \frac{A}{x} + \frac{B}{(x+3)}$$

3. Para hallar A y B, se sustituye la x por el valor de las raíces:

$$3x+2 = A(x+3) + Bx$$

$$x = 0 \rightarrow 2 = 3A \rightarrow A = \frac{2}{3}$$

$$x = -3 \rightarrow -7 = -3B \rightarrow B = \frac{7}{3}$$

4. Se escribe la integral como suma de integrales:

$$\int \frac{3x+2}{x^2+3x} dx = \int \frac{2/3}{x} dx + \int \frac{7/3}{(x+3)} dx = \frac{2}{3} \ln|x| + \frac{7}{3} \ln|x+3| + C$$

17) $\int \frac{4x+3}{x^2-1} dx$

1. Se analizan las raíces del denominador: $x^2 - 1 = 0$; $x = \sqrt{1} \rightarrow x = 1$; $x = -1$

2. Se escribe la descomposición en fracciones simples:

$$\frac{4x+3}{x^2-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$$

3. Para hallar A y B, se sustituye la x por el valor de las raíces:

$$4x+3 = A(x+1) + B(x-1)$$

$$x=1 \rightarrow 7 = 2A \rightarrow A = \frac{7}{2}$$

$$x=-1 \rightarrow -1 = -2B \rightarrow B = \frac{1}{2}$$

4. Se escribe la integral como suma de integrales:

$$\int \frac{4x+3}{x^2-1} dx = \int \frac{7/2}{(x-1)} dx + \int \frac{1/2}{(x+1)} dx = \frac{7}{2} \ln|x-1| + \frac{1}{2} \ln|x+1| + C$$

18) $\int \frac{3x^2}{x^2+6x+9} dx$

1. Al ser el grado del polinomio del numerador igual que el del denominador, debemos realizar la división de estos, $3x^2 : (x^2 + 6x + 9) \rightarrow$ y obtenemos como cociente 3 y como resto $-18x - 27$

a. Obtenemos la siguiente integral: $\int 3dx + \int \frac{-18x-27}{x^2+6x+9} dx \rightarrow 3x + \int \frac{-18x-27}{x^2+6x+9} dx$

2. Con la nueva integral obtenida, realizamos el procedimiento habitual para integrales racionales:

a. Igualamos el polinomio del divisor a cero: $x^2 + 6x + 9 = 0 \rightarrow x = -3$ (doble)

b. Con las soluciones obtenidas calculamos: $\frac{-18x-27}{x^2+6x+9} = \frac{A}{x+3} + \frac{B}{(x+3)^2} \rightarrow$

$$-18x - 27 = A(x+3)^2 + B(x+3)$$

Sustituimos x por un valor cualquiera:

$$x = -2 \rightarrow 9 = A + B ; \quad x = -1 \rightarrow -9 = 4A + 4B$$

$$\text{Resolvemos el sistema de ecuaciones obtenido: } A = -\frac{27}{2}; \quad B = \frac{45}{2}$$

3. Gracias a este procedimiento hemos obtenido:

$$3x + \int \frac{-27}{x+3} dx + \int \frac{45}{(x+3)^2} dx = 3x - \frac{27}{2} \int \frac{dx}{x+3} + \frac{45}{2} \int \frac{dx}{(x+3)^2} =$$

$$3x - \frac{27}{2} \ln|x+3| + \frac{45}{2} \cdot \frac{(x+3)^{-1}}{-1} = 3x - \frac{27}{2} \ln|x+3| - \frac{45}{2} \cdot \frac{1}{x+3} + C$$

$$19) \int \frac{x+2}{x^2-5x+6} dx$$

1. Como el grado del polinomio del denominador es superior al del polinomio del numerador, utilizamos el procedimiento habitual para estos casos:

- Igualemos el polinomio del denominador a cero: $x^2 - 5x + 6 = 0 \rightarrow x_1 = 3; x_2 = 2$
- Con las soluciones obtenidas calculamos:

$$\frac{x+2}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2} \rightarrow x+2 = A(x-2) + B(x-3)$$

Sustituimos x por un valor cualquiera:

$$x = 2 \rightarrow 4 = -B \rightarrow B = -4; \quad x = 3 \rightarrow 5 = A$$

2. Gracias a este procedimiento obtenemos:

$$\int \frac{5}{x-3} dx + \int \frac{-4}{x-2} dx = 5\ln|x-3| - 4\ln|x-2| + C$$

$$20) \int \frac{3x-2}{x^2-4x+4} dx$$

1. Como el grado del polinomio del denominador es superior al del polinomio del numerador, utilizamos el procedimiento habitual para estos casos:

- Igualemos el polinomio del divisor a cero: $x^2 - 4x + 4 = 0 \rightarrow x = 2$ (doble)
- Con las soluciones obtenidas calculamos:

$$\frac{3x-2}{x^2-4x+4} = \frac{A}{x-2} + \frac{B}{(x-2)^2} \rightarrow 3x-2 = A(x-2)^2 + B(x-2)$$

Sustituimos x por un valor cualquiera: $x = 3 \rightarrow 7 = A + B; x = 1 \rightarrow 1 = A - B$

Resolvemos el sistema de ecuaciones obtenido: $A = 4; B = 3$

2. Gracias a este procedimiento obtenemos:

$$\int \frac{4}{x-2} dx + \int \frac{3}{(x-2)^2} dx = 4\ln|x-2| + 3 \cdot \frac{(x-2)^{-1}}{-1} = 4\ln|x-2| - \frac{3}{x-2} + C$$

$$21) \int \frac{3x+1}{x^3-4x^2+3x} dx$$

$$\frac{3x+1}{x^3-4x^2+3x} = \frac{3x+1}{x(x-3)(x-1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x-1} \Rightarrow \frac{3x+1}{x(x-3)(x-1)} = \frac{A(x-3)(x-1) + Bx(x-1) + Cx(x-3)}{x(x-3)(x-1)}$$

$$3x + 1 = (Ax - 3A)(x - 1) + Bx^2 - Bx + Cx^2 - 3Cx$$

$$3x + 1 = Ax^2 - Ax - 3Ax + 3A + Bx^2 - Bx + Cx^2 - 3Cx$$

$$3x + 1 = (A+B+C)x^2 - (4A + B + 3C)x + 3A$$

$$A + B + C = 0$$

$$-(4A + B + 3C) = 3$$

$$3A = 1 \Rightarrow A = \frac{1}{3}$$

$$\begin{cases} \frac{1}{3} + B + C = 0 \\ \frac{4}{3} + B + 3C = 3 \end{cases} \Rightarrow \begin{cases} B + C = -\frac{1}{3} \\ B + 3C = 3 - \frac{4}{3} \end{cases} \Rightarrow \begin{cases} B + C = -\frac{1}{3} \\ -B - 3C = 3 + \frac{4}{3} \end{cases} \Rightarrow \begin{cases} B + C = -\frac{1}{3} \\ -2C = 4 \end{cases} \Rightarrow C = \frac{4}{-2} \Rightarrow C = -2$$

$$B = -A - C = -\frac{1}{3} - (-2) \Rightarrow B = \frac{5}{3}$$

$$\int \frac{3x+1}{x^3-4x^2+3x} dx = \int \frac{\frac{1}{3}}{x} dx + \int \frac{\frac{5}{3}}{x-3} dx + \int \frac{-2}{x-1} dx = \frac{1}{3} \int \frac{dx}{x} + \frac{5}{3} \int \frac{dx}{x-3} - 2 \int \frac{dx}{x-1} =$$

$$= \frac{1}{3} \ln(x) + \frac{5}{3} \ln(x-3) - 2 \ln(x-1) + C$$

22) $\int \frac{2x^2-1}{x^2+3x+2} dx$ efectuando la división obtenemos, $C(x) = 2$; $R(x) = -6x - 5$, de donde,

$$\frac{D}{d} = C + \frac{R}{d} \Rightarrow \int \frac{2x^2-1}{x^2+3x+2} dx = \int 2dx + \int \frac{-(6x+5)}{x^2+3x+2} dx = 2 \int dx - \int \frac{6x+5}{x^2+3x+2} dx = I_1 + I_2$$

$$I_1 = 2 \int dx = 2x + C$$

$$I_2 = \int \frac{6x+5}{x^2+3x+2} \Rightarrow \frac{6x+5}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$\frac{6x+5}{(x+2)(x+1)} = \frac{A(x+1)+B(x+2)}{(x+2)(x+1)} \Rightarrow 6x+5 = Ax + A + Bx + 2B$$

$$6x+5 = (A+B)x + A + 2B$$

$$A+B=6$$

$$A+2B=5$$

$$B=-1$$

$$A=7$$

$$\int \frac{7}{x+2} dx + \int \frac{-1}{x+1} dx = 7 \int \frac{dx}{x+2} - \int \frac{dx}{x+1} = 7 \ln(x+2) - \ln(x+1) + C$$

$$\int \frac{2x^2-1}{x^2+3x+2} dx = 2x - 7 \ln(x+2) - \ln(x+1) + C$$

23) $\int \frac{x-1}{x^2-4x+4} dx$

$$\frac{x-1}{x^2-4x+4} = \frac{x-1}{(x-2)^2} = \frac{A}{(x-2)^2} + \frac{B}{(x-2)} \Rightarrow \frac{x-1}{(x-2)^2} = \frac{A+B(x-2)}{(x-2)^2} \Rightarrow x-1 = A + Bx - 2B$$

$$B=1$$

$$A - 2B = -1 \Rightarrow A - 2(1) = -1 \Rightarrow A = 1$$

$$\int \frac{x-1}{x^2-4x+4} dx = \int \frac{1}{(x-2)^2} dx + \int \frac{1}{x-2} dx = I_1 + I_2$$

$$I_1 \Rightarrow \int \frac{dx}{(x-2)^2} \quad \left\{ \begin{array}{l} z = x - 2 \\ dz = dx \end{array} \right. = \int \frac{dz}{z^2} = \int z^{-2} dz = \frac{z^{-1}}{-1} + C = -\frac{1}{z} + C = -\frac{1}{x-2} + C$$

$$I_2 \Rightarrow \int \frac{dx}{x-2} \quad \left\{ \begin{array}{l} z = x - 2 \\ dz = dx \end{array} \right. = \int \frac{dz}{z} = \ln(z) + C = \ln|x-2| + C$$

$$\int \frac{x-1}{x^2-4x+4} dx = -\frac{1}{x-2} + \ln|x-2| + C$$

$$24) \int \frac{3x-1}{x^2+6x+9} dx = \int \frac{3x-1}{(x+3)(x+3)} dx = \int \frac{3x-1}{(x+3)^2} dx$$

$$\frac{3x-1}{(x+3)^2} = \frac{A}{(x+3)^2} + \frac{B}{(x+3)} \rightarrow \frac{3x-1}{(x+3)^2} = \frac{A+B(x+3)}{(x+3)^2}$$

$$3x - 1 = A + B(x + 3)$$

$$x = -3; 3 \cdot (-3) - 1 = A + B(-3 + 3) \rightarrow A = 10$$

$$x = 3; 3 \cdot 3 - 1 = -10 + B(3 + 3) \rightarrow 8 = -10 + 6B \rightarrow B = \frac{18}{6} = 3$$

$$\int \frac{3x-1}{x^2+6x+9} dx = \int \frac{-10}{(x+3)^2} dx + \int \frac{3}{(x+3)} dx = -10 \int \frac{1}{(x+3)^2} dx + 3 \int \frac{1}{(x+3)} dx =$$

$$= -10 \int (x+3)^{-2} dx + 3 \int \frac{1}{(x+3)} dx = -10 \cdot \frac{(x+3)^{-1}}{-1} + 3 \ln|x+3| + C = \frac{10}{(x+3)} + 3 \ln|x+3| + C$$

9. Halla el valor de las siguientes integrales definidas

$$1) \int_1^3 \frac{dx}{2x} = \int_1^3 \frac{1}{2x} dx = \frac{1}{2} \int_1^3 \frac{1}{x} dx = \frac{1}{2} \ln(|x|) \Big|_1^3 = \frac{1}{2} \ln(3) - \frac{1}{2} \ln(1) = \frac{1}{2} \ln(3)$$

$$2) \int_2^3 \frac{x}{x^2-1} dx = \int_2^3 \frac{1}{2t} dt = \frac{1}{2} \int_2^3 \frac{1}{t} dt = \frac{1}{2} \ln(|t|) \Big|_2^3 = \frac{1}{2} \ln(|x^2-1|) \Big|_2^3 = \frac{1}{2} \ln(3^2-1) - \frac{1}{2} \ln(2^2-1) = \frac{1}{2} \ln\left(\frac{8}{3}\right)$$

$$3) \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin(x) dx = -\cos(x) \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} = -\cos\left(\frac{5\pi}{4}\right) - \left(-\cos\left(\frac{\pi}{4}\right)\right) = \frac{-1+\sqrt{2}}{2}$$

$$4) \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin(3x) dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin(t)}{3} dt = \frac{1}{3} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin(t) dt = \frac{1}{3} (-\cos(t)) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{1}{3} (-\cos(3x)) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} =$$

$$-\frac{\cos(3x)}{3} \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = -\frac{\cos\left(3 \times \frac{\pi}{4}\right)}{3} - \left(-\frac{\cos\left(3 \times \frac{\pi}{6}\right)}{3}\right) = \frac{\sqrt{2}}{6}$$

$$5) \int_{-4}^4 |x| dx = \int_{-4}^0 (-x) dx + \int_0^4 x dx = -\frac{x^2}{2} \Big|_{-4}^0 + \frac{x^2}{2} \Big|_0^4 = 8 + 8 = 16$$

$$6) \int_{-1}^1 \left(3x^2 - 2x + \frac{1}{2}\right) dx = \int_{-1}^1 3x^2 dx - \int_{-1}^1 2x dx + \int_{-1}^1 \frac{1}{2} dx = \left(x^3 - x^2 + \frac{1}{2}x\right) \Big|_{-1}^1 = \left(1^3 - 1^2 + \frac{1}{2} \cdot 1\right) - \left((-1)^3 - (-1)^2 + \frac{1}{2} \cdot (-1)\right) = 3$$

$$7) \int_{-1}^2 \left(\frac{2}{x+2} - \frac{3}{x-3}\right) dx = \int_{-1}^2 \frac{2}{x+2} dx - \int_{-1}^2 \frac{3}{x-3} dx = (2 \ln(|x+2|) - 3 \ln(|x-3|)) \Big|_{-1}^2 = 10 \ln(2)$$

$$8) \int_{-2}^2 \left(\frac{3a}{5} - \frac{x}{2}\right) dx = \int_{-2}^2 \frac{3a}{5} dx - \int_{-2}^2 \frac{x}{2} dx = \left(\frac{3ax}{5} - \frac{x^2}{4}\right) \Big|_{-2}^2 = \frac{3a \cdot 2}{5} - \frac{2^2}{4} - \left(\frac{3a \cdot (-2)}{5} - \frac{(-2)^2}{4}\right) = \frac{12}{5}a$$

$$9) \int_2^3 \frac{1}{x \cdot (\ln x)^3} dx$$

$$\int \frac{1}{x \cdot (\ln x)^3} dx = \int (\ln x)^{-3} \cdot \frac{1}{x} dx = \frac{(\ln x)^{-2}}{-2} = -\frac{1}{2 \cdot (\ln x)^2} + C$$

$$\int_2^3 \frac{1}{x \cdot (\ln x)^3} dx = -\frac{1}{2 \cdot (\ln x)^2} \Big|_2^3 = -\frac{1}{2 \cdot (\ln 3)^2} + \frac{1}{2 \cdot (\ln 2)^2} = -0,414 + 1,04 = 0,626$$

$$10) \int_{-2}^0 \left(e^{2x} + \frac{3}{e^{3x}}\right) dx$$

$$\int \left(e^{2x} + \frac{3}{e^{3x}}\right) dx = \int e^{2x} dx + \int e^{-3x} \cdot 3 dx = \frac{1}{2} \int e^{2x} \cdot 2 dx - \int e^{-3x} \cdot (-3) dx$$

$$= \frac{1}{2} e^{2x} - e^{-3x} + C$$

$$\int_{-2}^0 \left(e^{2x} + \frac{3}{e^{3x}}\right) dx = \frac{1}{2} e^{2x} - e^{-3x} \Big|_{-2}^0 = \left(\frac{1}{2} e^{2 \cdot 0} - e^{-3 \cdot 0}\right) - \left(\frac{1}{2} e^{2 \cdot (-2)} - e^{-3 \cdot (-2)}\right) = 403,91$$

$$11) \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)^2 dx$$

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)^2 dx = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin^2 x - 2 \sin x \cos x + \cos^2 x) dx =$$

$$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1 - \cos^2 x - 2 \sin x \cos x + \cos^2 x) dx = (x - \sin^2 x) \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} =$$

$$= \left[\frac{5\pi}{4} - \sin^2 \frac{5\pi}{4}\right] - \left[\frac{\pi}{4} - \sin^2 \frac{\pi}{4}\right] = [5,23] - [0,79] = 4,44$$

10. Halla el valor de b para que se cumpla $\int_{-1}^b (2bx - 3x^2) dx = -12$.

1. Se resuelve la integral con la incógnita b:

$$\int_{-1}^b (2bx - 3x^2) dx = 2b \frac{x^2}{2} - 3 \frac{x^3}{3} \Big|_{-1}^b = bx^2 - x^3 \Big|_{-1}^b$$

2. Sustituimos los límites de integración:

$$(b \cdot b^2 - b^3) - (b \cdot (-1)^2 - (-1)^3) = b^3 - b^3 - b - 1 = -b - 1$$

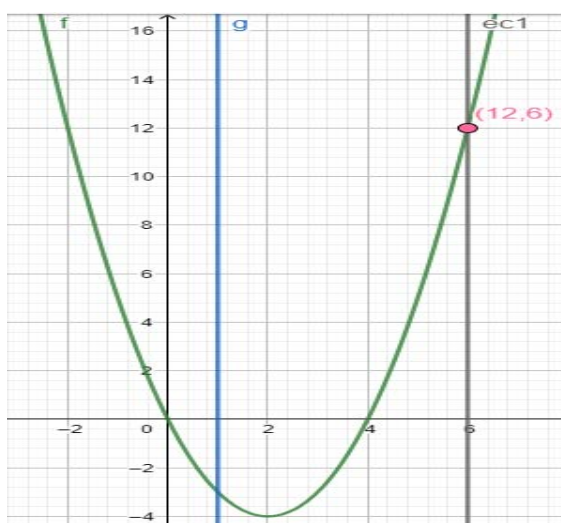
3. Igualamos el resultado a -12:

$$-b - 1 = -12 \rightarrow -b = -12 + 1 \rightarrow -b = -11 \rightarrow b = 11$$

Resultado: b=11

11. Halla el área entre la función $f(x) = x^2 - 4x$, el eje de abscisas, y las rectas $x=1$ y $x=6$.

1. Hacemos el gráfico:



2. Hallamos los cortes con el eje x de la función:

$$f(x) = x^2 - 4x \rightarrow x^2 - 4x = 0 \rightarrow x(x - 4) = 0 \rightarrow x_1 = 0, \quad x_2 = 4$$

3. Hallamos el área de las dos zonas de áreas obtenidas; de $x=1$ a $x=4$, y de $x=4$ a $x=6$:

$$\int_1^4 (x^2 - 4x) dx = \left[\frac{x^3}{3} - 4 \frac{x^2}{2} \right]_1^4 = \left[\frac{x^3}{3} - 2x^2 \right]_1^4 = \left(\frac{4^3}{3} - 2 \cdot 4^2 \right) - \left(\frac{1^3}{3} - 2 \cdot 1^2 \right) = -9$$

Como es un área tomamos su valor positivo, 9.

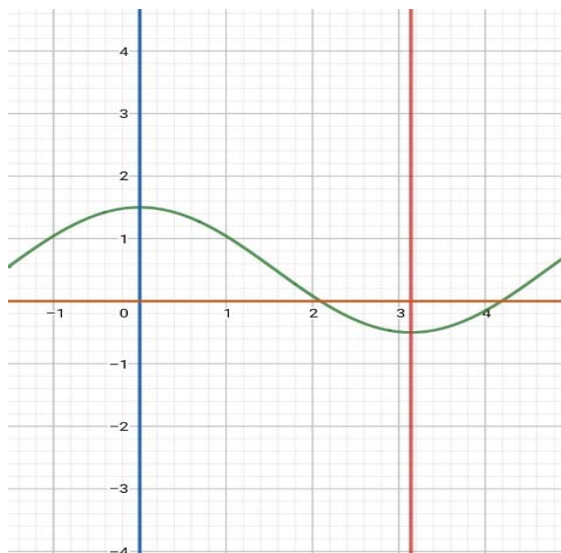
$$\int_4^6 (x^2 - 4x) dx = \left[\frac{x^3}{3} - 4 \frac{x^2}{2} \right]_4^6 = \left[\frac{x^3}{3} - 2x^2 \right]_4^6 = \left(\frac{6^3}{3} - 2 \cdot 6^2 \right) - \left(\frac{4^3}{3} - 2 \cdot 4^2 \right) = \frac{32}{3}$$

4. Sumamos ambas áreas:

$$\frac{32}{3} + (9) = \frac{59}{3} \text{ u. a.}$$

Resultado: El área es $\frac{59}{3}$ u.a.

12. Halla el área limitada por la función $f(x) = 0,5 + \cos x$, el eje de abscisas y las rectas $x = 0$ y $x = \pi$.



$$a=0$$

$$b=\pi$$

$$c = 0,5 + \cos x = 0; \quad \cos x = -0,5$$

$$x = \arccos(-0,5) = \frac{2\pi}{3}$$

$$A_t = A_1 + A_2;$$

$$A_1 = \int_a^c f(x) dx = \int_0^{\frac{2\pi}{3}} (0,5 + \cos x) dx = \int_0^{\frac{2\pi}{3}} 0,5 dx + \int_0^{\frac{2\pi}{3}} \cos x dx = (0,5x + \sin x) \Big|_0^{\frac{2\pi}{3}} =$$

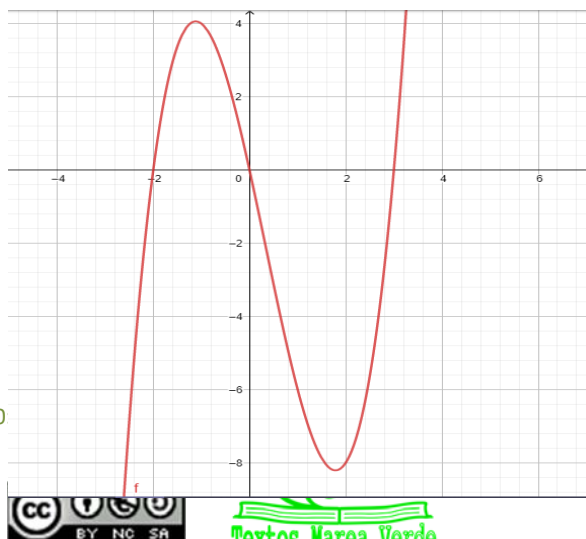
$$= \left[0,5 \left(\frac{2\pi}{3} \right) + \sin \left(\frac{2\pi}{3} \right) \right] - [0,5(0) + \sin(0)] = 1,08 - 0 = 1,08 u^2$$

$$A_2 = \left| \int_c^b f(x) dx \right| = \left| \int_{\frac{2\pi}{3}}^{\pi} (0,5 + \cos x) dx \right| = \left| (0,5x + \sin x) \Big|_{\frac{2\pi}{3}}^{\pi} \right| = \left| [0,5(\pi) + \sin(\pi)] - \left[0,5 \left(\frac{2\pi}{3} \right) + \sin \left(\frac{2\pi}{3} \right) \right] \right| = 0,55$$

$$A_t = 1,08 + 0,55 = 1,63 u^2$$

13. Halla el área de la región limitada por la función $f(x) = x^3 - x^2 - 6x$ y el eje de abscisas.

1. Hacemos el gráfico:



2. Hallamos los cortes con el eje x de la función:

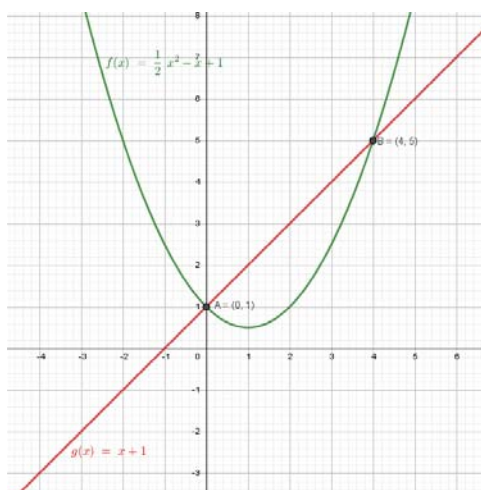
$$f(x) = x^3 - x^2 - 6x \rightarrow x^3 - x^2 - 6x = 0 \rightarrow x(x^2 - x - 6) = 0 \rightarrow x_1 = 0, x_2 = -2, x_3 = 3$$

3. Hallamos el área de las dos zonas obtenidas; de $x=-2$ a $x=0$, y de $x=0$ a $x=3$:

$$\begin{aligned} \int_{-2}^0 (x^3 - x^2 - 6x) dx - \int_0^3 (x^3 - x^2 - 6x) dx = \\ \left[\frac{x^4}{4} - \frac{x^3}{3} - 3x^2 \right]_{-2}^0 - \left(\frac{x^4}{4} - \frac{x^3}{3} - 3x^2 \right) \Big|_0^3 = \\ \left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - 3 \cdot (-2)^2 \right) - \left(\frac{3^4}{4} - \frac{3^3}{3} - 3 \cdot 3^2 \right) = 10,42 u^2 \end{aligned}$$

Resultado: El área es $\frac{125}{12}$ u.a.

14. Calcula el área de la porción del plano que limitan las curvas $y = \frac{1}{2}x^2 - x + 1$ e $y - x - 1 = 0$



$$\text{Área} = \int_0^4 \left[(x+1) - \left(\frac{1}{2}x^2 - x + 1 \right) \right] dx = \int_0^4 \left(\frac{1}{2}x^2 + 2x \right) dx = \left(\frac{1}{2} \cdot \frac{x^3}{3} + x^2 \right) \Big|_0^4 = \frac{80}{3} u. a.$$

15. Halla el área delimitada por las gráficas:

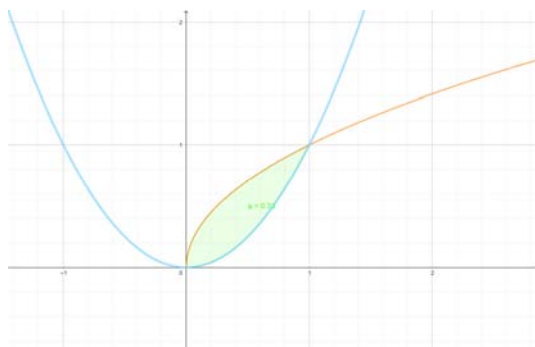
a) $f(x) = \sqrt{x}$ y $g(x) = x^2$

Igualamos $f(x)$ y $g(x)$ para hallar los puntos de corte:

$$\sqrt{x} = x^2 ; (\sqrt{x})^2 = (x^2)^2 ; x = x^4 ; x^4 - x = 0 ; x(x^3 - 1) = 0$$

$$x = 0$$

$$(x^3 - 1) = 0 ; x^3 = 1 ; x = \sqrt[3]{1} ; x = 1$$



Los puntos de corte son $x=0$ y $x=1$.

$$\begin{aligned} \text{Área} &= \int_0^1 (f(x) - g(x)) dx = \int_0^1 (\sqrt{x} - x^2) dx = \int_0^1 (x^{1/2} - x^2) dx = \left[\frac{2\sqrt{x^3}}{3} - \frac{x^3}{3} \right]_0^1 = \frac{2\sqrt{x^3} - x^3}{3} \Big|_0^1 = \\ &= \left(\frac{2\sqrt{1^3} - 1^3}{3} \right) - \left(\frac{2\sqrt{0^3} - 0^3}{3} \right) = \left(\frac{1}{3} \right) - (0) = \frac{1}{3} \text{ u. a.} \end{aligned}$$

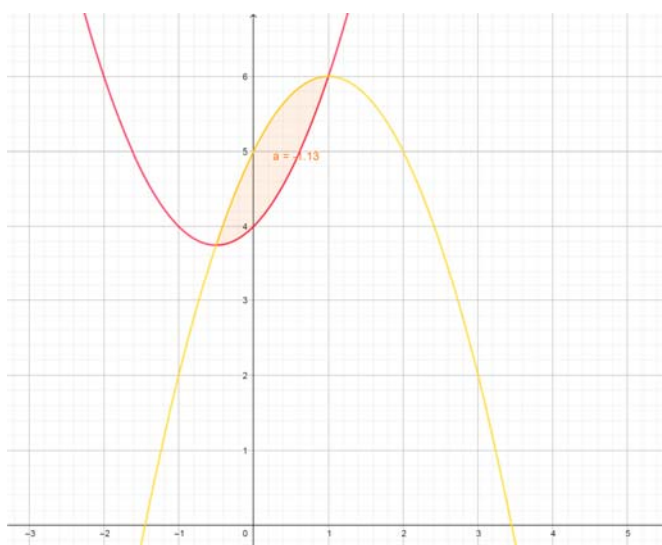
b) $f(x) = x^2 + x + 4$ y $g(x) = -x^2 + 2x + 5$

Igualemos $f(x)$ y $g(x)$ para hallar los puntos de corte:

$$x^2 + x + 4 = -x^2 + 2x + 5 \quad ; \quad 2x^2 - x - 1 = 0 \quad ; \quad x = 1 \quad ; \quad x = -\frac{1}{2}$$

Los puntos de corte son $x=1$ y $x=-\frac{1}{2}$

$$\begin{aligned} \text{Área} &= \left| \int_{-\frac{1}{2}}^1 (f(x) - g(x)) dx \right| = \left| \int_{-\frac{1}{2}}^1 ((x^2 + x + 4) - (-x^2 + 2x + 5)) dx \right| = \left| \int_{-\frac{1}{2}}^1 (x^2 + x + 4 + x^2 - 2x - 5) dx \right| \\ &= \left| \int_{-\frac{1}{2}}^1 (2x^2 - x - 1) dx \right| = \left| \left[\frac{2x^3}{3} - \frac{x^2}{2} - x \right]_{-\frac{1}{2}}^1 \right| = \left| \left(\frac{2 \cdot 1^3}{3} - \frac{1^2}{2} - 1 \right) - \left(\frac{2 \cdot (-\frac{1}{2})^3}{3} - \frac{(-\frac{1}{2})^2}{2} - \left(-\frac{1}{2} \right) \right) \right| = \\ &= \left| \left(-\frac{5}{6} \right) - \left(\frac{7}{24} \right) \right| = \left| -\frac{5}{6} - \frac{7}{24} \right| = \left| -\frac{9}{8} \right| = \frac{9}{8} \text{ u. a.} = 1,125 \text{ u. a.} \end{aligned}$$



Ejercicios Autoevaluación

- 1) Los valores de a, b y c para los que $F(x) = ax^3 - be^x + c \sin x$ es una primitiva de la función $f(x) = 3x^2 - 7e^x + 5 \cos x$ son:

$$F(x) = ax^3 - be^x + c \sin x$$

$$f(x) = 3x^2 - 7e^x + 5 \cos x$$

$$F'(x) = f(x)$$

$$F'(x) = 3ax^2 - be^x + c \cos x$$

$$a=1 \quad b=7 \quad c=5$$

La respuesta correcta es la b)

- 2) La integral indefinida $\int x\sqrt{2x^2 + 3} dx$ vale:

$$\int x\sqrt{2x^2 + 3} dx = \int x(2x^2 + 3)^{\frac{1}{2}} dx = \frac{1}{4} \int 4x(2x^2 + 3)^{\frac{1}{2}} dx =$$

$$\frac{1}{4} \cdot \frac{(2x^2+3)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{\sqrt{(2x^2+3)^3}}{6} + C$$

La respuesta correcta es la b)

- 3) La integral $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

$$\text{hacemos el cambio } t = \sin^2 x \quad dt = 2 \sin x \cos x dx = \sin 2x dx$$

$$\cos^4 x = (\cos^2 x)^2 = (1 - \sin^2 x)^2 = (1 - t)^2 = 1 - 2t + t^2$$

$$\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx = \int \frac{dt}{t^2 + 1 - 2t + t^2} = \int \frac{1}{2t^2 + 1 - 2t} dt = \int \frac{1}{2(t^2 + \frac{1}{2} - t)} dt =$$

$$\int \frac{1}{2(t^2 - t + \frac{1}{4})} dt = \int \frac{1}{2(t^2 - t + \frac{1}{4})} dt = \int \frac{1}{2((t - \frac{1}{2})^2 + \frac{1}{4})} dt = \frac{1}{2} \int \frac{1}{(t - \frac{1}{2})^2 + \frac{1}{4}} dt =$$

$$\frac{1}{2} \cdot \frac{1}{\frac{1}{2}} \arctan\left(\frac{t - \frac{1}{2}}{\frac{1}{2}}\right) = \frac{1}{2} \cdot \frac{1}{\frac{1}{2}} \arctan\left(\frac{\sin^2 x - \frac{1}{2}}{\frac{1}{2}}\right) = -\arctan(\cos 2x) + C$$

La respuesta correcta a este apartado es la d)

- 4) Al integrar por partes $\int \frac{x \cdot e^{\arcsen x}}{\sqrt{1-x^2}} dx$ se obtiene:

$$x = \text{senu}, \quad u = \arcsen x, \quad du = \frac{1}{\sqrt{1-x^2}} dx$$

$$\int e^u \sin u \, du = -e^u \cos u - \int (-e^u \cos u) du = -e^u \cos u - \left(- \int e^u \cos u \, du \right)$$

$$= -e^u \cos u - \left(- \left(e^u \sin u - \int e^u \sin u \, du \right) \right)$$

$$\text{Por lo tanto } \int e^u \sin u \, du = -e^u \cos u - (e^u \sin u - \int e^u \sin u \, du)$$

$$\text{Despejamos } \int e^u \sin u \, du = -\frac{e^u \cos u}{2} + \frac{e^u \sin u}{2} \text{ sustituimos en } u = \arcsen x$$

$$-\frac{e^{\arcsen x} \cos(\arcsen x)}{2} + \frac{e^{\arcsen x} \sin(\arcsen x)}{2} \text{ simplificamos} = -\frac{1}{2} e^{\arcsen x} (\sqrt{1-x^2} - x) + C$$

La respuesta a este apartado es la d)

5) La integral $\int \frac{2x+2}{x^2+4x+13} dx$ vale

$$\int \frac{2x+2+4-4}{x^2+4x+13} dx = \int \left(\frac{2x+4}{x^2+4x+13} - \frac{2}{x^2+4x+13} \right) dx = \int \frac{(2x+4)}{x^2+4x+13} dx - \int \frac{2}{(x+2)^2+9} dx =$$

$$= \ln(x^2 + 4x + 13) - \frac{2}{3} \arctan \frac{x+2}{3} + C$$

La respuesta correcta a este apartado es la a)

6) La integral $\int \frac{dx}{\sin^2 x \cos^2 x}$ vale

$t = \operatorname{tg} x$	$\sin x = \frac{t}{\sqrt{t^2+1}}$	$\sin^2 x = \frac{t^2}{t^2+1}$
$dx = \frac{dt}{1+t^2}$	$\cos x = \frac{1}{\sqrt{t^2+1}}$	$\cos^2 x = \frac{1}{t^2+1}$

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{\frac{dt}{1+t^2}}{\frac{t^2}{t^2+1} \cdot \frac{1}{t^2+1}} = \int \frac{t^2+1}{t^2} dt = \int \left(1 + \frac{1}{t^2} \right) dt = t - \frac{1}{t} + C = \operatorname{tg} x - \operatorname{cotg} x + C$$

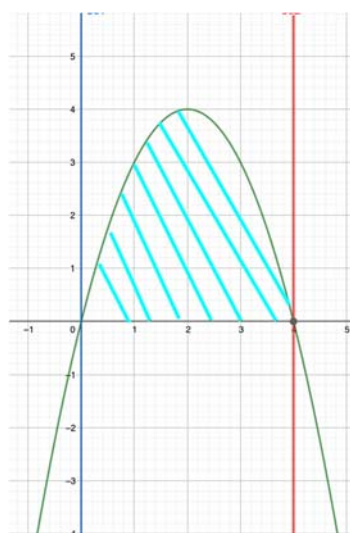
La respuesta a este apartado es la d)

7) La integral definida $\int_0^\pi \cos x dx$ vale:

$$\int_0^\pi \cos x dx = \sin x \Big|_0^\pi = \sin \pi - (\sin 0) = 0$$

La respuesta correcta es la c)

8) Para hallar el área comprendida entre la función $f(x) = -x^2 + 4x$, el eje de abscisas y las rectas $x=0$ y $x=4$, debemos representar dicha función y ver el área que comprende:



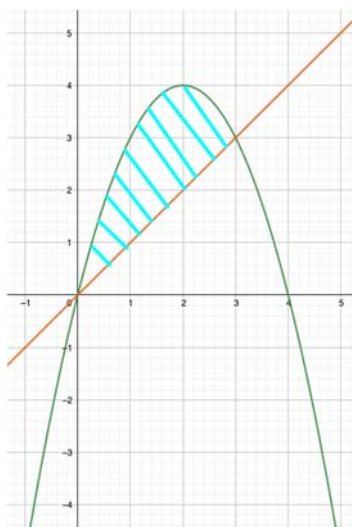
Una vez que tenemos la gráfica, y vemos donde corta la función con el eje y con las rectas, comenzamos a aplicar la regla de Barrow para obtener el área.

$$\int_0^4 (-x^2 + 4x) dx = -\frac{x^3}{3} + \frac{4x^2}{2} \Big|_0^4$$

$$\left(-\frac{4^3}{3} + \frac{4 \cdot 4^2}{2} \right) - (0) = \frac{32}{3}$$

La respuesta correcta es la b)

- 9) Para hallar el área comprendida entre las funciones $f(x) = -x^2 + 4x$ y $g(x) = x$, debemos representar ambas funciones y ver el área que comprenden:



Una vez que tenemos la gráfica, y vemos el dónde corta $f(x)$ con $g(x)$, debemos sacar los puntos de corte y, una vez hallados comenzamos a aplicar la regla de Barrow para obtener el área.

Puntos de corte: Para hallarlos debemos igualar las funciones y despejar la incógnita "x".

$$-x^2 + 4x = x$$

$$0 = x^2 - 3x$$

$$0 = x(x - 3)$$

$$x = 0$$

$$x = 3$$

Ahora ya podemos aplicar la regla de Barrow:

$$\int_0^3 [(-x^2 + 4x) - (x)] dx = \int_0^3 (-x^2 + 3x) dx = -\frac{x^3}{3} + \frac{3x^2}{2} \Big|_0^3$$

$$\left(-\frac{3^3}{3} + \frac{3 \cdot 3^2}{2} \right) - (0) = \frac{9}{2}$$

La respuesta correcta es la a)

10) El volumen del sólido de revolución generado por $y = x^2$, entre 0 y 2, al girar en torno al eje de abscisas es:

$$V = \pi \int_0^2 (x^2)^2 dx = \pi \left(\frac{x^5}{5} \right) \Big|_0^2 = \pi \frac{32}{5}$$

La respuesta a este apartado es la d)